Modern Physics

Length contraction

Length contraction: $L = \frac{L_o}{\gamma}$

Problem 1.- The distance from New York to Los Angeles is about 5000 km. How much shorter than 5000 km is the distance according to car travelers going at 60 mph?
The correction is of course very small in this case because of the very low velocity compared to the speed of light.

Solution: The correction is given by the equation:

$$\Delta x = L_o - \frac{L_o}{\gamma} = L_o - L_o\sqrt{1 - v^2/c^2} \approx L_o - L_o\left(1 - v^2/2c^2\right) = \frac{L_o v^2}{2c^2}$$

With the values of the problem:

$$v = 60\frac{\text{miles}}{\text{h}}\left(\frac{1609m}{1mile}\right)\left(\frac{1h}{3600s}\right) = 26.8m/s$$

And $L_o = 5\times10^6 m$

$$\Delta x = \frac{(5\times10^6)(26.8)^2}{2(3\times10^8)^2} = 20\,\text{nm}$$

Problem 2.- When neutrons are free (not being part of a nucleus) they are unstable with a half-life of 608 s. A source of fast neutrons emits a pulse of 1000 neutrons at v = 0.98c. Find how many will be left after traveling 1.79×10^{11} m.

Solution: First, let us calculate the length traveled from the point of view of the neutrons:

$$L = L_o/\gamma, \quad \text{but}\ \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{1}{\sqrt{1 - \frac{(0.98c)^2}{c^2}}} = \frac{1}{\sqrt{1 - 0.98^2}} = 5.025\text{, so:}$$

$$L = \frac{1.79\times10^{11}\text{m}}{5.025} = 3.56\times10^{10}\text{m}$$

Now, we can find the time needed to travel this distance from the point of view of the neutrons:

$$t = \frac{L}{0.98c} = \frac{3.56\times10^{10}\text{m}}{0.98(3\times10^{8}\text{m/s})} = 121\text{s}$$

Finally, we can find how many neutrons are left by using the decay equation:

$$N = N_o 2^{-\frac{t}{t_{1/2}}} = 1000\times2^{-\frac{121\text{s}}{608\text{s}}} = 871$$

Problem 3.- Why would you say that it is not a good idea to try the muon experiment on the ISS (international space station)?

Solution: muon production happens in the atmosphere, which is below the ISS.

Problem 4.- An object has a length of 6 nm at rest, but it is moving at 60% the speed of light in the direction of its length. How long does it appear?

(A) 1nm (B) 10nm (C) 7.5nm (D) 4.8nm (E) 3.6nm

Solution: we use the length contraction equation:

$$L = \frac{L_o}{\gamma} = \frac{6nm}{\frac{1}{\sqrt{1-0.6^2}}} = 4.8nm$$

Answer: **D**

Problem 5.- A proton is accelerated so much that the length of the accelerator seems to be only 0.8 times its proper length. Calculate the velocity of the proton.

Solution: Since $L = 0.8L_o \rightarrow \gamma = \frac{1}{0.8}$, so:

$$\frac{1}{\sqrt{1-\beta^2}} = \frac{1}{0.8} \rightarrow 1-\beta^2 = 0.64 \rightarrow \beta^2 = 0.36 \rightarrow \beta = 0.6$$

So, the speed of the particle is 0.6c

Problem 6.- A meter stick with a speed of 0.8*c* moves past an observer. In the observer's reference frame, how long does it take the stick to pass the observer?

(A) 1.6 ns
(B) 2.5 ns
(C) 4.2 ns
(D) 6.9 ns
(E) 8.3 ns

Solution: The length of the meter stick in the frame of the observer is: $L = \frac{1m}{\gamma} = 0.6m$

And the time is: $t = \frac{0.6m}{v} = \frac{0.6m}{0.8c} = \frac{0.6}{0.8 \times 3 \times 10^8} =$ **2.5 ns** **answer (B)**

Problem 7.- Two spaceships approach Earth with equal speeds, as measured by an observer on Earth, but from opposite directions. A meter stick on one spaceship is measured to be 60 cm long by an occupant of the other spaceship. What is the speed of each spaceship, as measured by the observer on Earth?

(A) $0.4c$
(B) $0.5c$
(C) $0.6c$
(D) $0.7c$
(E) $0.8c$

Solution: The length of the meter stick in the frame of the observer is: $L = \frac{1m}{\gamma} = 0.6m$

Which means that $\gamma = 1.666$ and the velocity from the point of view of the observer is:

$$\gamma = 1.666 \rightarrow \frac{1}{\sqrt{1-\beta^2}} = 1.6666 \rightarrow \beta = 0.8$$

This is the velocity of spacecraft 2 with respect to 1, in the frame of reference of spacecraft two, so if v is the speed of spacecraft 1 in the frame of reference of the Earth then $u_x = -v$ and we just learned that $u_x' = -0.8c$

$$u'_x = \frac{u_x - v}{1 - \frac{vu'_x}{c^2}} \rightarrow -0.8c = \frac{-v\text{-}v}{1 - \frac{v(\text{-}v)}{c^2}} \rightarrow 0.8 = \frac{2\beta}{1+\beta^2} \rightarrow 0.8\beta^2 - 2\beta + 0.8 = 0$$

There are two solutions, but only one is acceptable: $\beta = 0.5$ **answer (B)**

Problem 8.- Estimate the speed of a beam of particles whose half-life is 10^{-8} s in their own rest frame if in average they travel 30 meters in the laboratory frame before decaying.