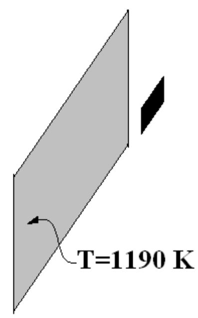
Modern Physics

Blackbody radiation

Stefan Boltzmann law for a blackbody: $\frac{Power}{Area} = \sigma T^4$, where $\sigma = 5.67\times10^{-8}\,\frac{\mathrm{W}}{\mathrm{m^2K^4}}$

Wien's law for a blackbody: $\lambda_{\text{max-intensity}}T = 2.898\times10^{-3}\,\mathrm{mK}$

Problem 1.- Calculate equilibrium temperature of a flat disk located in front of an infinite plane that is kept at a temperature of 1190 K.

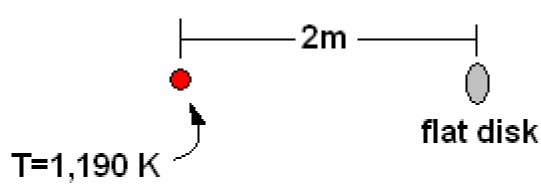


Solution: The object is in equilibrium, so the radiation emitted is equal to the absorbed, but emission happens on both sides, while absorption only on one, so:

$$Radiation\ Absorbed = Area\sigma(1190K)^4$$
$$Radiation\ Emitted = 2\times Area\sigma T^4$$

$$\to Area\sigma(1190K)^4 = 2\times Area\sigma T^4$$
$$\to T = \frac{1190}{\sqrt[4]{2}} = \mathbf{1000\ K}$$

Problem 2.- Calculate the equilibrium temperature of a small flat disk located 2m away from a spherical source of radiation kept at a temperature of 1,190 K. The source behaves like a blackbody and has a radius of 0.125m



Solution: Let us first find the total amount of power emitted by the small blackbody:

$$Power = 4\pi(0.125\mathrm{m})^2\sigma(1{,}190\mathrm{K})^4$$

This power will be equally distributed over a sphere of radius 2m, so the intensity where the flat disk is will be:

$$I = \frac{Power}{Area} = \frac{4\pi(0.125\text{m})^2\sigma(1{,}190\text{K})^4}{4\pi(2\text{m})^2}$$

The disk will absorb on one side, but emit on both, so the equilibrium equation is:

$$\frac{4\pi(0.125\text{m})^2\sigma(1{,}190\text{K})^4}{4\pi(2\text{m})^2} \times \pi r^2 = \sigma T^4 \times 2\pi r^2$$

Solving for T:

$$T = 1{,}190\text{K}\sqrt[4]{\frac{0.125^2}{8}} = \mathbf{250\ K}$$

Problem 3.- Polaris (the "North Star") is a trinary system where the brightest star has a surface temperature estimated at 7200K. Calculate the wavelength at which its maximum radiation intensity occurs. Assume that radiation is described by the blackbody equations.

Solution: We use Wien's law to get the wavelength:

$$\lambda_{\text{max-intensity}} T = 2.898\times10^{-3}\text{mK} \rightarrow \lambda_{\text{max-intensity}} = \frac{2.898\times10^{-3}\text{mK}}{7200\text{K}} = \mathbf{402\ nm}$$

That means that the brightest star of the system should look blue.

Problem 3a.- The spectrum of the microwave background radiation (which is present everywhere in our universe) can be closely approximated by the radiation of a blackbody at T=2.7 K. At what wavelength does it have the maximum intensity?

Solution: To find the wavelength we use Wien's law:

$$\lambda_{\text{max}} = \frac{2.898\times10^{-3}\text{mK}}{2.7\text{K}} = \mathbf{1.07\ mm}$$

Problem 4.- If it were possible to reach any temperature you want in an incandescent light bulb, what temperature would you choose to make it more efficient?

Solution: At the usual temperature of an incandescent bulb, the highest intensity is in the infrared. If you could reach the temperature of the sun's surface, you would have most of the radiation in the visible part of the spectrum.

Problem 5.- The second harmonic generator of an yttrium-aluminum-garnet (YAG) laser emits 100mJ pulses of green light (wavelength = 532nm). Calculate the number of photons emitted per pulse.

Solution: The energy of a single photon is:

$$E = \frac{hc}{\lambda} = \frac{(6.62\times10^{-34}\,Js)(3\times10^{8}\,m/s)}{532\times10^{-9}\,m} = 3.79\times10^{-19}\,J$$

So the number of photons in a 100mJ pulse is

$$\#of\ \ photons = \frac{0.1J}{3.79\times10^{-19}\,J} = \mathbf{2.68\times10^{17}}$$

Problem 6.- Consider that the sun with a radius R_{sun} can be approximated as a blackbody object with a surface temperature T_{sun}. A spherical planet behaves as a blackbody and is located at a distance r_{orbit} from the center of the sun. Assume $r_{orbit} >> R_{sun}$ to simplify the problem.

i) Calculate the surface temperature of the planet assuming it is in thermal equilibrium and there are no other sources of heat but the radiation from the sun.

ii) Based on that result, how much would be the temperature of the Earth, considering $R_{sun} = 6.96\times10^{8}\,m$, $r_{orbit} = 1.5\times10^{11}\,m$ and $T_{sun} = 5800K$

Solution: The total amount of radiation that the sun emits is:

$$Radiation_{sun} = \sigma T_{sun}^{4}\left(4\pi R_{sun}^{2}\right),$$

Where the quantity in parenthesis is the surface area of the sun. This power is equally distributed in all directions, so at the distance of the planet orbit the intensity is:

$$I = \frac{\sigma T_{sun}^{4}\left(4\pi R_{sun}^{2}\right)}{4\pi r_{orbit}^{2}} = \frac{\sigma T_{sun}^{4} R_{sun}^{2}}{r_{orbit}^{2}}$$

If $r_{orbit} >> R_{sun}$ the planet presents an area that we can approximate to that of a circle (πR_{planet}^{2}), so the planet absorbs:

$$Radiation_{absorbed} = I\left(\pi R_{planet}^{2}\right) = \frac{\pi\sigma T_{sun}^{4} R_{sun}^{2} R_{planet}^{2}}{r_{orbit}^{2}}$$

But the planet emits power in all possible directions. Assuming the temperature of the planet is T_{planet}, it emits:

$$Radiation_{planet} = \sigma T_{planet}^4 \left(4\pi R_{planet}^2\right)$$

If the planet is in equilibrium the amount of radiation absorbed has to be equal to the radiation emitted, so:

$$\frac{\pi\sigma T_{Sun}^4 R_{Sun}^2 R_{planet}^2}{r_{orbit}^2} = \sigma T_{planet}^4 \left(4\pi R_{planet}^2\right) \rightarrow T_{planet} = T_{Sun}\sqrt{\frac{R_{sun}}{2r_{orbit}}}$$

ii) Based on that result and the data given:

$$T_{planet} = 5800K\sqrt{\frac{6.96\times10^8 m}{2(1.5\times10^{11} m)}} = \mathbf{279\ K}$$

Notice that the result does not depend on the radius of the planet. You can apply the equation to other objects such as comets, other planets or spherical spacecraft.

Problem 7.- Estimate the power radiated by a basketball and a human being.

Solution: *Power radiated by a basketball*:

The power per unit area is given by the Stefan-Boltzmann law:

$$R(T) = \varepsilon\sigma T^4$$

We can assume T to be 20ºC, which means T=293 K. For the emissivity we will use 1, with the understanding that in reality it is less than that and our calculation will be only an upper bound.

$$R(T) = (1)(5.67\times10^{-8}\ W/m^2K^4)(293K)^4 = 418\ W/m^2$$

To get the power we should multiply this radiation times the area of the basketball (whose radius is 0.124 m):

$$Power = 4\pi r^2 R(T) = 4(3.14159)(0.124m)^2\times418W/m^2 = \mathbf{81\ W}$$

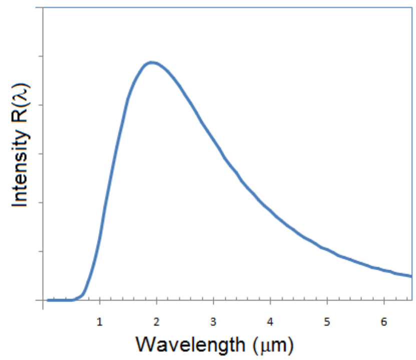
Power radiated by a human being:

We can solve this problem in a similar fashion. For the area of a person we can assume it as a cylinder that is 1.6 m high and with a radius of 0.25 m. Again, we can make the emissivity equal to 1, understanding that the result will be an upper bound. For T we will take 37°C or 310K.

$$R(T) = (1)(5.67\times10^{-8}\ W/m^2K^4)(310K)^4 = 524\ W/m^2$$

$\text{Power} = 2\pi rLR(T) = 2(3.14159)(0.25\text{m})(1.6\text{m})524\text{W/m}^2 = \mathbf{1{,}320\ W}$

Problem 8.- The intensity of blackbody radiation from a solid object as a function of wavelength λ is shown in the figure. If the Wien constant is $2.9\times10^{-3}\,\text{mK}$, what is the approximate temperature of the object?



(A) 10 K

(B) 50 K

(C) 250 K

(D) 1,500 K

(E) 6,250 K

Solution: The graph shows a maximum in the intensity at a wavelength of about 2μm. Using this value in Wien's equation we get:

$$\lambda_{\text{max-intensity}}T = 2.9\times10^{-3}\,\text{mK} \rightarrow T = \frac{2.9\times10^{-3}\,\text{mK}}{2\times10^{-6}\,\text{m}} = \mathbf{1{,}450\ K}$$