Modern Physics

Boltzmann

Problem 1.- Find the probability of finding the hydrogen atom in the low-lying hyperfine excited state at liquid helium temperature (T=4.2 K).
Approximate the atom as a two-level system where the energy of the excited state is 9.46×10^{-25} J higher than the ground state and assume the degeneracies are 1 for the ground state and 3 for the first excited state.

Solution: According to Boltzmann the probability is:

$$P_2 = \frac{g_2 e^{-E_2/k_BT}}{\sum g_j e^{-E_j/k_BT}} = \frac{g_2 e^{-E_2/k_BT}}{g_1 e^{-E_1/k_BT} + g_2 e^{-E_2/k_BT}} = \frac{3e^{-E_2/k_BT}}{1e^{-E_1/k_BT} + 3e^{-E_2/k_BT}} = \frac{3e^{-(E_2-E_1)/k_BT}}{1+3e^{-(E_2-E_1)/k_BT}}$$

Notice that: $(E_2 - E_1)/k_BT = \dfrac{9.46\times10^{-25} J}{(1.38\times10^{-23} J/K)(4.2K)} = 0.016$

Which means that $e^{-(E_2-E_1)/k_BT} = 0.984$, so the probability is:

$$P_1 = \frac{3(0.984)}{1+3(0.984)} = \mathbf{0.747}$$

Notice that even at this low temperature, the excited state is very close to a simple ¾ probability, which would happen at high temperature.

Problem 1a.- Find the probability of finding the aluminum atom in the first excited state at room temperature (T=300 K). Approximate the atom as a two-level system where the energy of the first excited state is 0.0129 eV higher than the ground state and assume the degeneracies are 2 for the ground state and 4 for the first excited state.

Solution: According to Boltzmann the probability is:

$$P_2 = \frac{g_2 e^{-E_2/k_BT}}{\sum g_j e^{-E_j/k_BT}} = \frac{g_2 e^{-E_2/k_BT}}{g_1 e^{-E_1/k_BT} + g_2 e^{-E_2/k_BT}} = \frac{4e^{-E_2/k_BT}}{2e^{-E_1/k_BT} + 4e^{-E_2/k_BT}} = \frac{2e^{-(E_2-E_1)/k_BT}}{1+2e^{-(E_2-E_1)/k_BT}}$$

Notice that: $(E_2 - E_1)/k_BT = \dfrac{0.0129(1.6\times10^{-19} J)}{(1.38\times10^{-23} J/K)(300K)} = 0.499$

Which means that $e^{-(E_2-E_1)/k_BT} = 0.607$, so the probability is:

$$P_1 = \frac{2\times0.607}{1+2\times0.607} = \mathbf{0.548}$$