Modern Physics

Discovery of the electron

Problem 1.- What magnetic field is necessary to make a beam of 200eV electrons move in a circle of 0.15m radius? Do you need to consider relativity in this problem?

Solution: Let us find the speed of the electron using classical physics:

$$\frac{1}{2}mv^2 = 200eV = 200(1.6\times10^{-19}J) \rightarrow v = \sqrt{\frac{2\times200(1.6\times10^{-19}J)}{9.1\times10^{-31}kg}} = 8.39\times10^6\ \text{m/s},$$

Since this speed is less than 10% the speed of light, we do not need to use relativistic equations for a good approximation.
The magnetic force provides the centripetal acceleration in this problem:

$$\text{F}_{\text{magnetic}} = qvB = m\frac{v^2}{R} \rightarrow B = \frac{mv^2}{qvR} = \frac{2E}{qvR} = \frac{400eV}{e(8.39\times10^6 m/s)(0.15m)} = \mathbf{3.18\times10^{-4}\ T}$$

Problem 2.- Calculate the acceleration voltage so an electron reaches a speed of 1/15c.
Do you need to consider relativity in this problem?

Solution: Since the ratio v/c is 1/15, the relativistic correction is very small. However, let us calculate both ways:

$$\text{K.E.}_{\text{Classical}} = \frac{1}{2}\text{mv}^2 = \frac{1}{2}(9.1\times10^{-31}\text{kg})(2\times10^7\ \text{m/s})^2 = 1.82\times10^{-16}\text{J}$$

$$\text{K.E.}_{\text{Relativistic}} = (\gamma-1)\text{mc}^2 = \left(\frac{1}{\sqrt{1-\frac{\text{v}^2}{\text{c}^2}}}-1\right)\text{mc}^2 = \left(\frac{1}{\sqrt{1-\frac{1}{15^2}}}-1\right)(9.1\times10^{-31})(3\times10^8\text{m/s})^2 =$$

$1.826\times10^{-16}\text{J}$

Given the small difference, we can use the non-relativistic result. This kinetic energy has to come from the electrical potential energy, so:

$$(\text{Voltage})\times(\text{Charge}) = \text{K.E.} \qquad \rightarrow \text{Voltage} = \frac{1.82\times10^{-16}\text{J}}{1.6\times10^{-19}\text{C}} = \mathbf{1{,}140\ V}$$