Modern Physics

K and L lines

Threshold for K-lines production: $E_{threshold} = (Z-1)^2 E_o$ Where $E_o = 13.6eV$

Problem 1.- Calculate the wavelength of the K$_{alpha}$ line in gold and silver.

Solution: We can use the Bohr model for a transition:

$$\frac{hc}{\lambda} = Z^2 E_o \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right)$$

Here n goes from 2 to 1. However, we need to use $Z-1$ instead of Z, because the remaining electron shields the nucleus with one negative charge.

$$\frac{hc}{\lambda} = (Z-1)^2 E_o \left(\frac{1}{1^2} - \frac{1}{2^2} \right) = (Z-1)^2 E_o \left(\frac{3}{4} \right) \rightarrow \lambda = \frac{4hc}{3(Z-1)^2 E_o}$$

For Gold: $\lambda = \frac{4(4.14\times10^{-15} eVs)(3\times10^8 m/s)}{3(79-1)^2(13.6eV)} =$ **20 pm**

For Silver: $\lambda = \frac{4(4.14\times10^{-15} eVs)(3\times10^8 m/s)}{3(47-1)^2(13.6eV)} =$ **57.5 pm**

Problem 1a.- What is the wavelength of the K$_{alpha}$ line produced by bombarding W?

Solution: The energy of the photon of a K-alpha line corresponds to the transition between n=2 and n=1. We use the Bohr energy equation, but we replace Z for Z-1 due to the electron already present in the first shell. So:

$$\frac{hc}{\lambda} = \frac{(Z-1)^2}{1^2} E_o - \frac{(Z-1)^2}{2^2} E_o = \frac{3(Z-1)^2}{4} E_o \rightarrow \lambda = \frac{4hc}{3(Z-1)^2 E_o}$$

$\lambda = \frac{4(4.135\times10^{-15} eVs)(3\times10^8 m/s)}{3(74-1)^2(13.6eV)} =$ **22.8 pm**

Problem 1b.-What wavelength are the K lines in a copper target (Z_{Cu}=29).

Solution: The energy of this line corresponds to the transition of an electron from n=2 to n=1 after a hole is left in the most interior orbital. So the equation is:

$$\frac{hc}{\lambda}=\left(\frac{1}{1^2}-\frac{1}{2^2}\right)13.6eV(29-1)^2 \rightarrow \lambda=\frac{hc}{(3/4)13.6eV(28)^2}=\mathbf{1.55\ \text{Å}}$$

Problem 2.- What is the difference in wavelength between the K_{alpha} emissions of gold and platinum?

Solution: We can use the equation below to find the frequencies and the wavelengths for the K-alpha lines as follows:

$$f_{K\alpha}=\frac{3cR}{4}(Z-1)^2 \rightarrow \lambda_{K\alpha}=\frac{c}{f_{K\alpha}}=\frac{4}{3R(Z-1)^2}$$

We know that Z_{Gold}=79 and $Z_{Platinum}$=78 so:

$$\lambda_{K\alpha-Pt}-\lambda_{K\alpha-Au}=\frac{4}{3R}\left[\frac{1}{77^2}-\frac{1}{78^2}\right]=5.22\times10^{-13}\text{ m}$$

Problem 3.- Determine if electron bombardment by 20keV electrons is enough to produce the K lines in a copper target (Z_{Cu}=29).

Solution: The threshold to remove the closes electron of the copper atom is:

$$E_{threshold}=(Z-1)^2E_o=(29-1)^2(13.6eV)=10{,}700\text{ eV}$$

Then 20,000 V is more than enough to produce the K lines in a copper target.