# Modern Physics

## Pair production and annihilation

**Problem 1.-** A photon strikes an electron of mass *m* that is initially at rest, creating an electron-positron pair. The photon is destroyed, and the positron and two electrons move off at equal speeds along the initial direction of the photon. The energy of the photon was:

(A) $mc^2$ (B) $2mc^2$ (C) $3mc^2$ (D) $4mc^2$ (E) $5mc^2$

**Solution:** we calculate the total energy before and after:

$$\mathrm{T}E_{before} = \mathrm{T}E_{after} \rightarrow p_{photon}c + mc^2 = 3\sqrt{p_{electron}{}^2c^2 + m^2c^4}$$

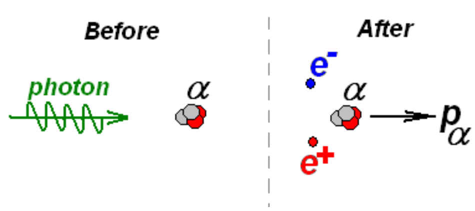
But notice that the momentum of the electron is 1/3 the momentum of the photon, so:

$$pc + mc^2 = 3\sqrt{\frac{p^2c^2}{9} + m^2c^4}$$ and solving for p we get:

$$p^2c^2 + 2pcmc^2 + m^2c^4 = p^2c^2 + 9m^2c^4 \rightarrow 2pcmc^2 = 8m^2c^4 \rightarrow p = 4mc$$ answer: **D**

**Problem 2.-** What is the minimum photon energy needed to create an electron-positron pair when a photon collides with an alpha particle?

**Solution:** If we want the minimum energy, we can assume that the alpha particle will take away the momentum of the incident photon and the electron-positron pair is created with zero linear momentum. This is a very close approximation taking into account how massive the alpha particle is compared to the electron-positron pair.




Conservation of linear momentum tells us that: $\frac{hf}{c} = p_\alpha$

And conservation of energy tells us that:

$$hf + m_\alpha c^2 = \sqrt{p_\alpha^2c^2 + m_\alpha^2c^4} + 2mc^2$$

Where *m* is the mass of the electron (or positron).

Replacing the first equation into the second: $hf + m_\alpha c^2 = \sqrt{h^2 f^2 + m_\alpha^2 c^4} + 2mc^2$
Which we can solve for *hf* as follows:

$$hf + m_\alpha c^2 - 2mc^2 = \sqrt{h^2 f^2 + m_\alpha^2 c^4}$$
$$\rightarrow h^2 f^2 + 2hf(m_\alpha c^2 - 2mc^2) + (m_\alpha c^2 - 2mc^2)^2 = h^2 f^2 + m_\alpha^2 c^4$$

Simplifying:

$$2hf(m_\alpha c^2 - 2mc^2) + (m_\alpha c^2 - 2mc^2)^2 = m_\alpha^2 c^4$$

$$hf = \frac{m_\alpha^2 c^4 - (m_\alpha c^2 - 2mc^2)^2}{2(m_\alpha c^2 - 2mc^2)} = \frac{\left[m_\alpha^2 - (m_\alpha - 2m)^2\right]c^2}{2(m_\alpha - 2m)} = \frac{2m(m_\alpha - m)c^2}{(m_\alpha - 2m)}$$

With the values:

Mass of the electron $=9.1\times10^{-31}$ kg
Mass of the alpha particle $=6.7\times10^{-27}$ kg

$$hf = 2mc^2\left[\frac{6.7\times10^{-27}kg - 9.1\times10^{-31}kg}{6.7\times10^{-27}kg - 2\times9.1\times10^{-31}kg}\right] = \mathbf{1.022\ MeV}\ [1.00014]$$

The correction to the energy of the photon is only 1.022 MeV[0.00014] = **140eV**

***An alternative solution:***
In a first approximation, let us assume that we create the pair at rest without any momentum transfer, so:
$hf = 2mc^2 = 1.022 MeV$
But the photon has momentum equal to: $p = \frac{hf}{c} = 2mc = \frac{1.022MeV}{c}$
If this is transferred to the alpha particle, the kinetic energy of the alpha particle is:

$$K.E. = \frac{p^2}{2m_{alpha}} = \frac{\left(\frac{1.022MeV}{c}\right)^2}{2m_{alpha}} = \frac{\left(\frac{1.022MeV}{c}\right)^2}{2\times3750MeV/c^2} = \mathbf{139\ eV}$$

The energy of the photon will need to be 139 eV greater than the 1.022MeV, a small correction indeed.

**Problem 3.-** What would be the minimum energy of a photon to create a proton-antiproton pair by collision with a heavy nucleus? (In your calculation, you can ignore the kinetic energy of the heavy nucleus).

**Solution**: To conserve energy the photon needs to have at least the rest energy of the two particles formed:

$$E \geq m_{proton}c^2 + m_{antiproton}c^2 = 2(1.67\times10^{-27}kg)(3\times10^8 m/s)^2 = 3.006\times10^{-10}\,\text{J}$$

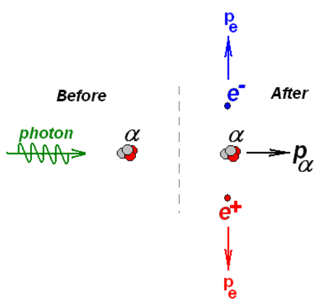
However, the momentum of the photon needs to be transferred to the nucleus. For example, let us say it is a U-238 nucleus:

The momentum is: $p = \dfrac{E}{c} = \dfrac{3\times10^{-10}}{3\times10^{8}} = 1\times10^{-18}\,kgm/s$

The kinetic energy in the nucleus: $KE = \dfrac{p^2}{2m_{nucleus}} = \dfrac{(1\times10^{-18})^2}{2\times238\times1.67\times10^{-27}} = 1.25\times10^{-12}\,J$

This is a small correction.

**Problem 4.-** A photon of energy 2MeV creates an electron-positron pair upon collision with an alpha particle which was originally at rest.
After the collision, the pair is scattered in opposite directions with identical momenta, and the alpha particle takes away the original momentum of the photon.
Calculate the kinetic energy of the alpha particle.
Take $m_{alpha}$=3,750MeV/$c^2$




**Solution:** The momentum of the photon is equal to $p = \dfrac{2MeV}{c}$. All this momentum is transferred to the alpha particle. If we ignore relativistic corrections, the kinetic energy of the alpha particle will be:

$$K.E._{non-relativistic} = \frac{p^2}{2m} = \frac{\left(\dfrac{2MeV}{c}\right)^2}{2\dfrac{3750MeV}{c^2}} = \mathbf{530\ eV}$$

This is such a small number compared to the rest energy of the alpha particle that we are justified in ignoring relativistic corrections.

**Problem 5.-** An electron moving towards the right with speed 0.5c collides with a positron that was moving towards the left with a speed of 0.5. The two particles disappear and emit two gamma rays. Calculate the energy of the gamma rays.

**Solution:** The total energy of the electron becomes a photon:

$$E = \frac{hc}{\lambda} = \gamma mc^2 = \frac{1}{\sqrt{1-0.5^2}}(511keV) = \mathbf{590\ keV}$$

**Problem 6.-** What is the wavelength of the pair of gamma rays created when an electron annihilates a positron in a PET scan?

**Solution:** This is a case of pair annihilation. Each photon will have the rest energy of the electron (or positron) so:

$$\frac{hc}{\lambda} = mc^2 \rightarrow \lambda = \frac{hc}{mc^2} = \frac{4.136\times10^{-15} eVs(3\times10^8 m/s)}{0.511\times10^6 eV} = \mathbf{2.42\ pm}$$