Modern Physics

Rutherford scattering

Impact parameter in Rutherford scattering: $b = \frac{Z_1 Z_2 e^2}{8\pi\varepsilon_o K.E.}\cot\left(\frac{\theta}{2}\right)$

Problem 1.- Calculate the impact parameter for an alpha particle scattered at 45º from a gold nucleus. The kinetic energy of the alpha particle is 5 MeV and neglect any recoil of the target.

Solution:

We can use the equation: $b = \frac{Z_1 Z_2 e^2}{8\pi\varepsilon_o K.E.}\cot\left(\frac{\theta}{2}\right)$

So:

$$b = \frac{(2)(79)(1.602\times10^{-19})^2}{8\pi(8.8542\times10^{-12})(5\times10^6\times1.602\times10^{-19})}\cot\left(\frac{45^\circ}{2}\right) = \mathbf{5.54\times10^{-14}\ m}$$

Problem 1a.- In the Rutherford experiment, calculate the impact parameter for an alpha particle scattered at 90º from a gold nucleus. Consider the kinetic energy of the alpha particle to be 7 MeV and neglect any recoil of the target.

Solution: We can use the equation: $b = \frac{Z_1 Z_2 e^2}{8\pi\varepsilon_o K.E.}\cot\left(\frac{\theta}{2}\right)$

So: $$b = \frac{(2)(79)(1.602\times10^{-19})^2}{8\pi(8.8542\times10^{-12})(7\times10^6\times1.602\times10^{-19})}\cot\left(\frac{90^\circ}{2}\right) = \mathbf{1.62\times10^{-14}\ m}$$

Problem 2.- Calculate the impact parameter for an alpha particle scattered at 45º from a gold nucleus. Consider the alpha particle moving at 3×10^6 m/s and neglect any recoil of the target. Is this result comparable to the size of the nucleus (larger, smaller, about the same)?

Solution: The impact parameter in Rutherford scattering is given by:

$$b = \frac{Z_1 Z_2 e^2}{8\pi\varepsilon_o K.E.}\cot\left(\frac{\theta}{2}\right)$$

And with the values of the problem:

$$b = \frac{(79)(2)(1.6\times10^{-19}C)^2}{8\pi(8.85\times10^{-12}F/m)\left(\frac{1}{2}(4\times1.67\times10^{-27}kg)(3\times10^6 m/s)^2\right)}\cot\left(\frac{45^\circ}{2}\right) = \mathbf{1.46\times10^{-12}m}$$

Notice that we used the non-relativistic equation for the kinetic energy, which is justified because the speed is only 1% of the speed of light.
The impact parameter is much larger than the nucleus.

Problem 3.- In a Rutherford scattering experiments you use 8MeV alpha particles and you detect 100,000 particles scattered at angles greater than 10º, how many do you expect to detect scattered at more than 20º?

Solution: According to the theory of Rutherford scattering, the fraction of particles is proportional to $\cot^2\left(\frac{\theta}{2}\right)$, so the ratio between particles scattered at more than 20º to particles scattered more than 10º will be:

$$\frac{N_{\theta>20^\circ}}{N_{\theta>10^\circ}} = \frac{\cot^2\left(\frac{20^\circ}{2}\right)}{\cot^2\left(\frac{10^\circ}{2}\right)} = \frac{\cot^2(10^\circ)}{\cot^2(5^\circ)} = 0.246$$

We expect **24,600** particles scattered more than 20º.

Problem 4.- You ran two almost identical Rutherford scattering experiments: same energy, same time, same angle, etc. The only difference is that you use copper foil and gold foil, but you keep the number of atoms per unit area the same. In the experiment with gold you detect 3,000 particles. How many do you detect with copper?

Solution: Notice that if everything is equal but the atomic number of the target, the Rutherford scattering equation says that the number of scattered particles will be proportional to the square of the atomic number. So the ratio of intensity between copper and gold is:

$$\frac{N_{Cu}}{N_{Au}} = \frac{29^2}{79^2} = 0.135$$

In the experiment with gold, you detect 3,000 particles, so you expect only 0.135(3000) = **405** particles with copper.

Problem 5.- In the Rutherford experiment, if the incoming alpha particle has enough energy and collides with the nucleus at a short impact parameter, it is possible that they will touch. At that point the distance between the particles will be the sum of the radii and nuclear forces will start playing a role in the collision, not only electromagnetic forces.

For a head on collision (where b=0) calculate the minimum energy of the alpha particle to touch a gold nucleus.

Repeat the calculation for an aluminum nucleus and for an incoming proton instead of an alpha particle.

Solution: The values that we need can be summarized as follows:

	radius		Charge				
proton	1.30E-15	m	1	e	=	1.602E-19	C
alpha particle	2.60E-15	m	2	e	=	3.204E-19	C
Gold nucleus	7.00E-15	m	79	e	=	1.266E-17	C
Aluminum Nucleus	3.60E-15	m	13	e	=	2.083E-18	C

To find the minimum kinetic energy of the alpha particle we assume that this kinetic energy is completely converted into potential energy at the point of contact and so:

$$\text{K.E.} = \frac{q_1 q_2}{4\pi\varepsilon_o (r_1 + r_2)}$$

Where q_1 and q_2 are the charges and r_1 and r_2 the radii. This calculation ignores energy transferred to the kinetic energy of the two particles as they recoil during the collision. This is reasonable if the target nucleus in the solid and behaves as a much larger mass.

For the collision with gold:

$$K.E. = 3.8 \times 10^{-12} J = 23.7 MeV$$

For the collision with aluminum:

$$K.E. = 9.68 \times 10^{-13} J = 6.06 MeV$$

b) For the proton collisions we follow the same procedure, finding:
For the collision with gold:

$$K.E. = 2.2 \times 10^{-12} J = 13.7 MeV$$

For the collision with aluminum:

$$K.E. = 6.12 \times 10^{-13} J = 3.83 MeV$$

Problem 6.- Calculate the cross section of gold for angles greater than 90º in the Rutherford experiment.

Solution: We can calculate the impact parameter for 90º and then find the cross section by calculating πb^2

$$b = \frac{Z_1 Z_2 e^2}{8\pi\varepsilon_o K.E.} \cot\left(\frac{\theta}{2}\right)$$

$$b = \frac{(2)(79)(1.602\times10^{-19})^2}{8\pi(8.8542\times10^{-12})(4.8\times10^6\times1.602\times10^{-19})}\cot\left(\frac{90^\circ}{2}\right) = 2.38\times10^{-14}\text{ m}$$

Cross section: $\pi b^2 = \mathbf{1.78\times10^{-27}\ m^2}$