

Modern Physics

Angular momentum

Problem 1.- An electron in a hydrogen atom is in the $L = 4$, state. Find the smallest angle the orbital angular momentum makes with the z -axis.

Solution: Angular momentum in quantum mechanics has some weird properties. If $L = 4$, it means that the value of L^2 is $4 \times (4+1)$ in units of \hbar squared, but this cannot be measured directly, rather, the projection on the z -axis will go from a minimum of -4 to $+4$, so the smallest angle with the z -axis is:

$$\theta = \cos^{-1}\left(\frac{4}{\sqrt{20}}\right) = 26.6^\circ$$

Problem 1a.- An electron in a hydrogen atom is in 3p state. Find the smallest angle the orbital angular momentum makes with the z -axis.

Solution: Like in the previous problem, if $L = 1$ (that is the case for the 3p state), it means that the value of L^2 is $1 \times (1+1)$ in units of \hbar squared and the projection on the z -axis can be -1 , 0 or $+1$, so the smallest angle with the z -axis is:

$$\theta = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = 45^\circ$$

Problem 2.- What is the magnitude of the spin momentum of an electron? (Express your answer in terms of \hbar).

Solution: Similar to orbital angular momentum, the spin angular momentum is $s = 1/2$, but the value of s^2 is $1/2 \times (1/2+1)$ in units of \hbar squared, so the magnitude is

$$|\vec{s}| = \sqrt{\frac{1}{2}\left(\frac{1}{2}+1\right)}\hbar = 0.866\hbar$$