Modern Physics

de Broglie Waves

Problem 1.-

a) Calculate the wavelength of a slow neutron (you can use non-relativistic equations) whose kinetic energy is 75 eV.

b) Calculate the wavelength of a fast neutron (you need to use relativistic equations) whose kinetic energy is 1.8 GeV.

Solution:

a) $$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mKE}} = \frac{6.62\times10^{-34}\,Js}{\sqrt{2\times1.67\times10^{-27}\,kg\times75\times1.6\times10^{-19}\,J}} = 3.31\,pm$$

b) $$\lambda = \frac{h}{p} = \frac{hc}{\sqrt{TE^2 - m^2c^4}} = \frac{4.14\times10^{-15}\,eVs\times3\times10^{8}\,m/s}{\sqrt{(1.8+0.94)^2 - 0.94^2}\times10^{9}\,eV} = 0.48\,fm$$

b) Using relativistic momentum:

$$(K.E.+mc^2)^2 = p^2c^2 + m^2c^4 \rightarrow p = \frac{\sqrt{(K.E.+mc^2)^2 - m^2c^4}}{c}$$

The de Broglie wavelength is:

$$\lambda = \frac{hc}{\sqrt{(K.E.+mc^2)^2 - m^2c^4}} = \frac{4.135\times10^{-15}\,eVs(3\times10^{8}\,m/s)}{\sqrt{(10^{12}\,eV + 939\times10^{6}\,eV)^2 - \left(939\times10^{6}\,eV\right)^2}} = \mathbf{1.24\ am}$$

Problem 2.- Calculate the theoretical maximum resolution (comparable to the de Broglie wavelength) of an electron microscope that uses electrons accelerated to 3MeV.

Solution: We can safely say that the resolution limit is of the same order of magnitude as the de Broglie wavelength of the electrons used.
Since the kinetic energy is larger than the rest energy of the electron, we should use relativity to get the momentum:

$$\mathrm{E}^2 = \mathrm{p}^2\mathrm{c}^2 + \mathrm{m}^2\mathrm{c}^4$$

In this equation, E is the total energy, kinetic plus energy at rest. Solving for p we get:

$$\mathrm{p} = \frac{\sqrt{\mathrm{E}^2 - \mathrm{m}^2\mathrm{c}^4}}{\mathrm{c}} = \frac{\sqrt{(\mathrm{K.E.} + \mathrm{mc}^2)^2 - \mathrm{m}^2\mathrm{c}^4}}{\mathrm{c}}$$

Plugging in the values of the problem:

$$p = \frac{\sqrt{(3\text{MeV} + 0.511\text{MeV})^2 - (0.511\text{MeV})^2}}{c} = \frac{3.474\text{MeV}}{c}$$

Now, the wavelength:

$$\lambda = \frac{h}{p} = \frac{4.1357\times10^{-15}\,\text{eVs}}{3.474\times10^{6}\,\text{eV}/3\times10^{8}\,\text{m/s}} = \mathbf{3.6\times10^{-13}m}$$

Problem 3.-Consider the argon atoms present in air. As an average, when are their de Broglie wavelengths shorter, in winter or summer?

Solution: The higher the temperature the higher the average momentum and so the shorter the wavelength, so the right answer is summer.

Problem 4.- The wavefunction of a free electron is given by:

$$\Psi(x,t) = C\sin\left(\frac{2\pi\ x}{1.54\ \text{nm}}\right)$$

Find its momentum in $\text{kg}\frac{\text{m}}{\text{s}}$

Solution: The wavefunction is a sine function, so the de Broglie wavelength is obtained directly for that function:

$$\lambda = 1.54nm$$

The momentum will be:

$$p = \frac{h}{\lambda} = \frac{6.625\times10^{-34}\,Js}{1.54\times10^{-9}\,m} = 4.3\times10^{-25}\,\text{kgm/s}$$

Problem 5.- The wavefunction of a free electron is given by:

$$\Psi(x,t) = A\sin(2.5\times10^{10}\,x)$$

Find its momentum, kinetic energy, and de Broglie wavelength.

Solution: We can re-write the wave function as follows:

$$\Psi(x,t) = \frac{A}{2i}\left(e^{2.5\times10^{10}\,xi} - e^{-2.5\times10^{10}\,xi}\right)$$

This is a linear superposition of two wave functions with opposite momentum, so the expected value is zero. However the absolute value is

$$p = \hbar k = \left(1.055\times10^{-34}\,\mathrm{Js}\right)\left(2.5\times10^{10}\,\mathrm{m}^{-1}\right) = 2.64\times10^{-24}\,\mathrm{kgm/s}$$

The kinetic energy is:

$$E = \frac{p^2}{2m} = \frac{\left(2.64\times10^{-24}\,\mathrm{kgm/s}\right)^2}{2(9.1\times10^{-31}\,\mathrm{kg})} = 3.83\times10^{-18}\,\mathrm{J} = 23.9\mathrm{eV}$$

We are justified in using the non-relativistic equation because the kinetic energy is very small compared to the rest energy of the electron (511 keV).

To get the de Broglie wavelength we identify the wavenumber:

$$\frac{2\pi}{\lambda} = k = 2.5\times10^{10}\,\mathrm{m}^{-1} \rightarrow \lambda = \frac{2\pi}{2.5\times10^{10}\,\mathrm{m}^{-1}} = 2.51\times10^{-10}\,\mathrm{m}$$

Problem 6.- A free particle with kinetic energy *E* and de Broglie wavelength λ enters a region in which it has potential energy *V*. What is the particle's new de Broglie wavelength?

(A) $\lambda\left(1+\frac{E}{V}\right)$ (B) $\lambda\left(1-\frac{E}{V}\right)$ (C) $\frac{\lambda}{1-\frac{E}{V}}$ (D) $\lambda\sqrt{1+\frac{E}{V}}$ (E) $\frac{\lambda}{\sqrt{1-\frac{E}{V}}}$

Solution: The kinetic energy will be E-V in the region and the new momentum will be smaller than before by a factor $(1\text{-}V/E)^{0.5}$, so the de Broglie wavelength will be longer by $(1\text{-}V/E)^{-0.5}$

Answer: **E**