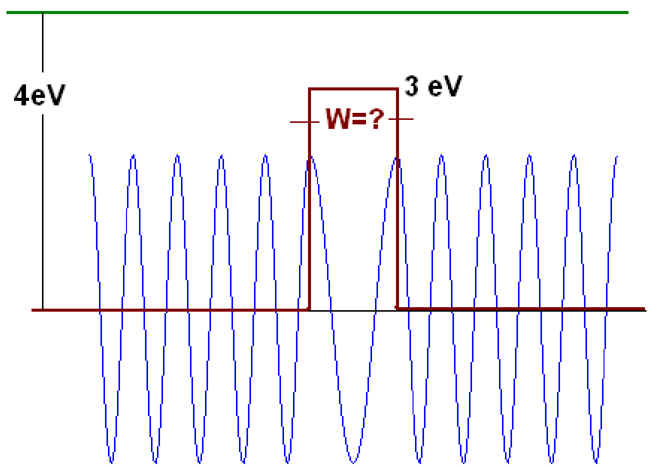
Modern Physics

Tunnel Effect

If $E > V_o$: $$T = \frac{1}{1+\frac{V_o^2\sin^2(kL)}{4E(E-V_o)}} \quad \text{where} \quad k = \frac{\sqrt{2m(E-V_o)}}{\hbar}$$

If $E < V_o$: $$T = \frac{1}{1+\frac{V_o^2\sinh^2(kL)}{4E(V_o-E)}} \quad \text{where} \quad k = \frac{\sqrt{2m(V_o-E)}}{\hbar}$$

Problem 1.- Suppose a beam of electrons of kinetic energy 4.0eV approaches a 3.0eV barrier. Calculate the width of the barrier if there is no reflection.



Solution: The condition for no reflection is: $W = \frac{n\lambda}{2}$, where n is an integer and λ is the de Broglie wavelength in the barrier. We also know that

$p = \frac{h}{\lambda}$, so $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK.E.}} = \frac{h}{\sqrt{2m(E-V)}}$, so the condition of no reflection becomes:

$$W = \frac{nh}{2\sqrt{2m(E-V)}}$$

With the values of the problem:

$$W = \frac{n \times 6.62\times10^{-34}\,Js}{2\sqrt{2(9.1\times10^{-31}\,kg)(1.6\times10^{-19}\,J)}} = \boldsymbol{n\ 6.13\text{Å}}$$

Problem 2.- Calculate the probability of an electron of kinetic energy 5eV to tunnel through a 6 eV barrier 1.5nm in width.

Solution: First, we calculate the wavenumber and then use the tunneling equation to find the probability:

$$k = \frac{\sqrt{2m(V_o - E)}}{\hbar} = \frac{\sqrt{2(9.1\times10^{31}kg)(1eV)}}{1.05\times10^{-34}Js} = \frac{\sqrt{2(9.1\times10^{31}kg)(1.6\times10^{-19}J)}}{1.05\times10^{-34}Js} = 5.14\times10^{9}\text{m}^{-1}$$

$$T = \frac{1}{1+\frac{V_o^2\sinh^2(kL)}{4E(V_o-E)}} = \frac{1}{1+\frac{(6eV)^2\sinh^2\left((5.14\times10^9m^{-1})(1.5\times10^{-9}m)\right)}{4\times5eV(6eV-5eV)}} = \mathbf{4.4\times10^{-7}}$$

So, only about one in two million particles will tunnel through the barrier.

Problem 2a.- Suppose a beam of electrons of kinetic energy 3.0eV approaches a 4.0eV barrier that has a width of 0.55nm. Calculate the probability of the electrons to tunnel to the other side.

Solution: First, we calculate the wavenumber and then use the tunneling equation to find the probability:

$$k = \frac{\sqrt{2m(V_o - E)}}{\hbar} = \frac{\sqrt{2(9.1\times10^{31}kg)(1eV)}}{1.05\times10^{-34}Js} = \frac{\sqrt{2(9.1\times10^{31}kg)(1.6\times10^{-19}J)}}{1.05\times10^{-34}Js} = 5.14\times10^{9}\text{m}^{-1}$$

$$T = \frac{1}{1+\frac{V_o^2\sinh^2(kL)}{4E(V_o-E)}} = \frac{1}{1+\frac{(4eV)^2\sinh^2\left((5.14\times10^9m^{-1})(0.55\times10^{-9}m)\right)}{4\times3eV(4eV-3eV)}} = \mathbf{0.0105}$$

So, just 1% of the particles will tunnel through the barrier.