Modern Physics

Uncertainty principle

Problem 1.- An ultra-fast laser produces pulses of UV light of 248nm wavelength and 9fs long.

What is the minimum uncertainty in the energy of the photons?
Give your answer in eV and as a percentage of the energy of the photon.

Solution: The energy of a photon of wavelength 248 nm is:

$$E = \frac{hc}{\lambda} = \frac{4.135\times10^{-15} eVs(3\times10^{8} m/s)}{248\times10^{-9} m} = 5.00 eV$$

The uncertainty in time should be roughly the length of the pulse, so 9 fs, and to find the uncertainty in the energy we use the Heisenberg principle for conjugated variables:

$$\Delta E \Delta t > \frac{\hbar}{2} \to \Delta E > \frac{\hbar}{2\Delta t} = \frac{0.658\times10^{-15} eVs}{2(9\times10^{-15} s)} = \mathbf{0.037\ eV}$$

This uncertainty in percentage is:

$$\frac{\Delta E}{E}\times100\% = \frac{0.037 eV}{5.00 eV}\times100\% = \mathbf{0.73\ \%}$$

Problem 2.- A second order transition in condensed matter can happen when a particle visits a forbidden state for a certain amount of time and then decays into a stable state. The time of the visit and the forbidden energy are conjugated variables. If you know the time is 1.3fs, how much would the forbidden energy be?

Solution: based on the uncertainty principle:

$$\Delta E \Delta t \geq \frac{\hbar}{2} \to \Delta E \geq \frac{\hbar}{2\Delta t} = \frac{1.05\times10^{-34} Js}{2(1.3\times10^{-15} s)} = \mathbf{4.0 \times 10^{-20}\ J}$$

Problem 3.- A tunable laser produces pulses of UV light in the 250nm to 400nm range. From the point of view of quantum mechanics what is the uncertainty in the energy of the photons if the pulses are 9ns long?

Solution: Since the pulse is spread over time, there is an uncertainty of the order of 9ns, this implies that the uncertainty in the energy will be at least the one given by the Heisenberg Principle:

$$\Delta E \Delta t \geq \frac{\hbar}{2} \to \Delta E \geq \frac{\hbar}{2\Delta t} = \frac{6.5821\times10^{-16} eVs}{2(9\times10^{-9} s)} = 3.66\times10^{-8} eV$$

This is a very small uncertainty, but this is only a lower bound, there are other restrictions in a real application that will make this limit very difficult to reach.

Problem 4.- What should be the uncertainty in the energy of a particle whose half life is 10^{36} years?

Solution: Since the energy and time are conjugated variables, we can calculate the uncertainty in the energy by using the relation:

$$\Delta E \Delta t \geq \frac{\hbar}{2} \rightarrow \Delta E \geq \frac{\hbar}{2\Delta t} = \frac{1.055\times10^{-34}\,\text{Js}}{2(10^{36}\,\text{year})\left(\dfrac{3.15\times10^{7}\,\text{s}}{1\text{year}}\right)} = 1.67\times10^{-78}\,\text{J}$$

This uncertainty is extremely small.

Problem 5.- Calculate the uncertainty in the position of a Helium atom if the uncertainty in its momentum is $\sqrt{6\text{m}_{\text{Helium}}\text{k}_{\text{B}}\text{T}}$, where $\text{m}_{\text{Helium}} = 4(1.67\times10^{-27}\,\text{kg})$, k_B is Boltzmann constant and T=4 K.