Modern Physics

Hydrogen atom

Problem 1.- What is the degeneracy of the n=4 shell of the hydrogen atom? Give your answer by listing the states that have the same energy.

Solution: Degeneracy $=2n^2=$**32**

The states are as follows:

n	*l*	*m*	*s*
4	0	0	1/2 or -1/2
4	1	-1	1/2 or -1/2
4	1	0	1/2 or -1/2
4	1	1	1/2 or -1/2
4	2	-2	1/2 or -1/2
4	2	-1	1/2 or -1/2
4	2	0	1/2 or -1/2
4	2	1	1/2 or -1/2
4	2	2	1/2 or -1/2
4	3	-3	1/2 or -1/2
4	3	-2	1/2 or -1/2
4	3	-1	1/2 or -1/2
4	3	0	1/2 or -1/2
4	3	1	1/2 or -1/2
4	3	2	1/2 or -1/2
4	3	3	1/2 or -1/2

Problem 2.- Where is the maximum probability of finding the electron of a hydrogen atom that has the quantum numbers $n=3,\ l=0,\ m=0,\ s=1/2$. Explain your reasoning.

Solution: Since the quantum number l is zero, the maximum probability is at $r=0$.

Problem 3.- What is the smallest value of the quantum number l if the angular momentum of the electron is less than 1° off the z-axis?

Solution: The minimum angle will happen when $L_z=l\hbar$ and then:

$$\cos\theta=\frac{l}{\sqrt{l(l+1)}}\rightarrow\cos^2\theta=\frac{l}{l+1}\rightarrow l=\frac{\cos^2\theta}{1-\cos^2\theta}=\frac{\cos^2 1^\circ}{1-\cos^2 1^\circ}=\mathbf{3283}$$

Problem 4.- What is the degeneracy of the n=7 shell of the hydrogen atom, ignoring magnetic fields.

Solution: The degeneracy is $2n^2$, so for n=7 it will be **98**.

Problem 5.- The wavefunction of the only electron of hydrogen in the second excited state and with quantum number L=0 and m_L=0 is given (in spherical coordinates) by the formula:

$$\psi = \frac{1}{4\sqrt{2\pi}a^{3/2}}\left[2-\frac{r}{a}\right]e^{-r/2a}$$, where "a" is the Bohr radius

Find the places where the probability of finding the electron is zero.

Solution: the exponential functions decays with the radius, and this is much faster than the linear function inside the brackets, so the wave function is zero at infinity.
The other possibility is that $r = 2a$, which is a sphere with radius $2a$.