Modern Physics

Rutherford model

Problem 1.- If an electron were orbiting a proton with a classical circular orbit of radius 1.2fm, what would be its kinetic energy? What would be its speed?

Solution: Recall that in the planetary model the kinetic energy is one half the absolute value of the potential energy, so:

$$\text{K.E.} = \frac{1}{2}\left|\frac{q_1 q_2}{4\pi\varepsilon_o r}\right|$$

so:

$$K.E. = \frac{(1.602C)^2}{8(3.14159)(8.85\times10^{-12}Fm)(1.2\times10^{-15}m)} = 9.61\times10^{-14}\text{ J} = 0.60\text{ MeV}$$

This energy is extremely high; it is comparable to the rest energy of the electron (its energy due to its mass mc^2).

A naïve calculation (without considering relativity) gives us:

$$v = \sqrt{2K.E./m} = \sqrt{2(9.61\times10^{-14}J)/(9.1\times10^{-31}kg)} = 4.60\times10^{8}\text{ m/s}$$

This is unreasonable because it is faster than the speed of light.

Problem 2.- If an electron is in the n=3 state and stays there for 10ns before decaying, how many rotations does it make in that time?

Solution: Recall that the period is given by $T = \frac{2\pi r}{v}$ and the period is:

$$T = \frac{2\pi r}{\left(\frac{n\hbar}{mr}\right)} = \frac{2\pi m r^2}{n\hbar} = \frac{2\pi m(n^2 a_o)^2}{n\hbar} = n^3\left[\frac{2\pi m a_o{}^2}{\hbar}\right]$$

Given that n=3 we get:

$$T = 3^3\left[\frac{2(3.14159)(9.1\times10^{-31}\text{kg})(0.529\times10^{-10}\text{m})^2}{1.05\times10^{-34}\text{Js}}\right] = 4.11\times10^{-11}\text{s}$$

The number of revolutions will be the time given divided by this period:

$$Number\ of\ revolutions = \frac{10^{-8}s}{4.11\times10^{-11}s} = 2.43\times10^{6}$$