Modern Physics

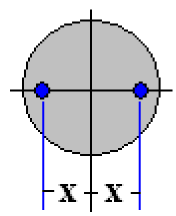
Thomson model

Magnetic force: $\vec{F}_{\text{magnetic}} = q\vec{v}\times\vec{B}$

Electric force: $\vec{F}_{\text{electric}} = q\vec{E}$

Mass of the electron: $m_e = 9.1\times10^{-31}kg$

Problem 1.- The plum pudding analogy hides the fact that Thomson thought the electrons were moving inside the atom. Nevertheless, let us consider a plum pudding model of helium: A spherical volume of radius R with homogeneously distributed positive charge = +2e and two electrons, with charge –e each one, at rest as shown in the figure:



What is the value "x" for the electrons to be in equilibrium?

Solution: There are two forces acting on each electron: The one due to the other electron and the one due to the charge inside the sphere of radius "x". Notice that the charge outside this radius does not contribute any net force.

- The force due to the other electron is repulsive and given by:

$$F_{electron} = \frac{1}{4\pi\varepsilon_o}\frac{e^2}{(2x)^2}$$

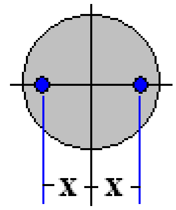
- The force due to the charge inside the sphere is given by:

$$F_{positive\ charge} = \frac{1}{4\pi\varepsilon_o}\frac{e\left(2e\frac{x^3}{R^3}\right)}{x^2}$$

Notice that the charge inside the sphere of radius "x" is only a fraction of the total positive charge. If the density of charge is uniform, this fraction is proportional to the radius cubed.
For equilibrium to happen:

$$\frac{1}{4\pi\varepsilon_o}\frac{e^2}{(2x)^2} = \frac{1}{4\pi\varepsilon_o}\frac{e\left(2e\frac{x^3}{R^3}\right)}{x^2} \rightarrow \frac{R^3}{2^2} = 2x^3 \rightarrow \boxed{x = \frac{R}{2}}$$

Problem 1a.- The plum pudding analogy hides the fact that Thomson thought the electrons were moving inside the atom. Nevertheless, let us consider a plum pudding model of a lithium ion (Li^+): A spherical volume of radius R with homogeneously distributed positive charge = +3e and two electrons, with charge –e each one, at rest as shown in the figure:



What is the value "x" for the electrons to be in equilibrium?

Solution: There are two forces acting on each electron: The one due to the other electron and the one due to the charge inside the sphere of radius "x". Notice that the charge outside this radius does not contribute any net force. So:

The force due to the other electron is repulsive and given by: $F_{electron} = \frac{1}{4\pi\varepsilon_o}\frac{e^2}{(2x)^2}$

The force due to the charge inside the sphere is given by:

$$F_{positive\,charge} = \frac{1}{4\pi\varepsilon_o}\frac{e\left(3e\frac{x^3}{R^3}\right)}{x^2}$$

Notice that the charge inside the sphere of radius "x" is only a fraction of the total positive charge. If the density of charge is uniform, this fraction is proportional to the radius cubed.
For equilibrium to happen:

$$\frac{1}{4\pi\varepsilon_o}\frac{e^2}{(2x)^2} = \frac{1}{4\pi\varepsilon_o}\frac{e\left(3e\frac{x^3}{R^3}\right)}{x^2} \rightarrow \frac{R^3}{12} = x^3 \rightarrow x = \frac{R}{\sqrt[3]{12}}$$

Problem 2.- In reproducing Thomson's experiment**,** what electric field is necessary to compensate the deflection of a beam of electrons moving at v=3×10^{6}m/s in a magnetic field of 0.15 T?

Solution: The two forces need to be equal, so:

$$Bqv = Eq \rightarrow E = Bv = 0.15T \times 3\times10^6 m/s = 4.5\times10^5 V/m$$

Problem 3.- What magnetic field is necessary to make a beam of 200eV-electrons move in a circle of 0.15m radius? Do you need to consider relativity in this problem?

Solution: Let us find the speed of the electron using classical physics:

$$\frac{1}{2}mv^2 = 200eV = 200(1.6\times10^{-19}J) \rightarrow v = \sqrt{\frac{2\times200(1.6\times10^{-19}J)}{9.1\times10^{-31}kg}} = 8.39\times10^6 \text{m/s},$$

Since this speed is less than 10% the speed of light, we do not need to use relativistic equations.

The magnetic force provides the centripetal acceleration in this problem:

$$\text{F}_{\text{magnetic}} = qvB = m\frac{v^2}{R} \rightarrow B = \frac{mv^2}{qvR} = \frac{2E}{qvR} = \frac{400eV}{e(8.39\times10^6 m/s)(0.15m)} = \mathbf{3.18\times10^{-4}\ T}$$

This field is easily achieved in the lab.