Modern Physics

Black Body Radiation

A) We understand that simple thermodynamic ideas (the zeroth law) imply that perfect emitters are also perfect absorbers of radiation. That is why a perfect emitter will look black at room temperature and that is why we talk about "black" body radiation.

B) A black body can be obtained by covering a surface with carbon soot, which is pitch black, but it won't be perfect. A better black body would be a hole on the side of a cavity. It does not matter what material the cavity is made of, it will be a universal blackbody, because the probability that a ray of light would be reflected backwards through the hole is very small, making it an excellent absorber, and so an excellent emitter too.

C) Inside the cavity there will be radiation in the way of standing electromagnetic waves. These waves can only have wavelengths that are twice the length of the cavity divided by an integer number. Those are the only standing waves allowed. The reason is that the electric field has to be zero at the walls; otherwise, the field will shake the electrons disturbing the wave.

D) It is easy to see that you can allow shorter and shorter wavelengths to be standing waves in the cavity. In principle, there is no limit to how short they can be.

E) There is a powerful law of statistical physics that tells you that each degree of freedom that can store energy will store energy. The average energy stored is controlled by the quantity k_BT where k_B is Boltzmann constant and T is the absolute temperature. You are probably familiar, for example, with the $1/2k_BT$ of energy stored in each translational degree of freedom of a free molecule. This law is sometimes called the "equipartition principle", it is not really a new law of physics, but a corollary of statistics applied to thermal physics.

F) If you apply the equipartition principle to the standing waves in the cavity and you assume that each mode (each standing wave) can have any continuous amount of energy (from zero to infinity) you would get a contribution proportional to k_BT for each mode, so the sum would be infinite.

G) A careful calculation that derives the intensity of the blackbody radiation assuming these classical ideas is called the Rayleigh-Jeans formula:

$$I = \frac{2\pi c k_B T}{\lambda^4}$$

This is wrong because it predicts a diverging intensity as the wavelength gets shorter. This is called the "**UV catastrophe**".

H) The UV catastrophe was solved by Max Planck. He assumed that the energy in each mode could only be an integer times a "quantum" of energy, given by a new constant of nature that he called "h" times the frequency of the wave:

$$E = nh\nu$$

where ν (the Greek letter "nu") is the frequency and n is 0, 1, 2…
If you assume this, the average energy stored in each mode is no longer proportional to k_BT, the average energy stored in the cavity becomes finite and the intensity of the blackbody radiation becomes Planck's radiation law, which agrees with experiments.

Wien's Law
This law is based on an experimental observation, but it can also be derived from Planck's radiation law. It tells you that the maximum radiation intensity will happen at a wavelength inversely proportional to the absolute temperature.

This law is used to estimate the temperature of the surface of stars (including our beloved sun) and in instruments such as:

<u>Pyro thermometer:</u> It uses a hot filament to match the color of an oven and measure its temperature without contact. Typically, the filament goes inside a telescope so you can point the lens to the oven and compare the colors easily. Then you change the temperature of the filament by increasing the current going through it. When you do not see the filament anymore because the background has the same color, you know they both are at the same temperature.

<u>Infrared cameras:</u> Most objects that surround us are at low temperatures, so they emit their maximum intensity in the infrared (the colors we see in cold objects are from *reflected* visible light, not the objects' own infrared radiation). But if you can measure this wavelength you can determine the object's temperature. Cameras that employ this technology are commercially available. They are used in finding leaks in the insulation of buildings, in the electric power industry to detect faulty insulators and in law enforcement to "see in the dark".

Total energy radiated by a black body.

A very important consequence of Planck's radiation law is that the total power radiated by a blackbody is proportional to T to the fourth power!!! It means that if you increase the temperature by a factor of two, the total power will increase 16 times.