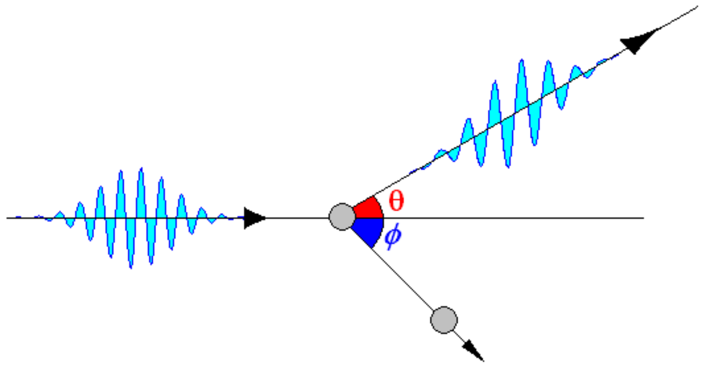
Modern Physics

Compton Effect



Conservation of momentum:

In x-direction: $$\frac{h}{\lambda} = \frac{h}{\lambda'}\cos\theta + p_e\cos\phi$$

In y-direction: $$0 = \frac{h}{\lambda'}\sin\theta - p_e\sin\phi$$

Eliminating the variable ϕ:

$$p_e^{\,2} = \left(\frac{h}{\lambda} - \frac{h}{\lambda'}\cos\theta\right)^2 + \frac{h^2}{\lambda'^2}\sin^2\theta = \frac{h^2}{\lambda^2} - \frac{2h^2}{\lambda\lambda'}\cos\theta + \frac{h^2}{\lambda'^2}$$

Conservation of energy:

$$\frac{hc}{\lambda} + mc^2 = \frac{hc}{\lambda'} + E_e \rightarrow E_e = \frac{hc}{\lambda} - \frac{hc}{\lambda'} + mc^2$$

But we know that: $E_e^{\,2} = p_e^{\,2}c^2 + m^2c^4$ so:

$$p_e^{\,2}c^2 + m^2c^4 = \left(\frac{hc}{\lambda} - \frac{hc}{\lambda'} - mc^2\right)^2 = \frac{h^2c^2}{\lambda^2} + \frac{h^2c^2}{\lambda'^2} + m^2c^4 - \frac{2h^2c^2}{\lambda\lambda'} + \frac{2hc}{\lambda}mc^2 - \frac{2hc}{\lambda'}mc^2$$

And replacing the equation for $p_e^{\,2}$:

$$\left(\frac{h^2}{\lambda^2} - \frac{2h^2}{\lambda\lambda'}\cos\theta + \frac{h^2}{\lambda'^2}\right)c^2 + m^2c^4 = \frac{h^2c^2}{\lambda^2} + \frac{h^2c^2}{\lambda'^2} + m^2c^4 - \frac{2h^2c^2}{\lambda\lambda'} + \frac{2hc}{\lambda}mc^2 - \frac{2hc}{\lambda'}mc^2$$

Simplifying:

$$\lambda' - \lambda = \frac{h}{mc}(1 - \cos\theta)$$