Modern Physics

Metals

We know that metals are good conductors of heat and electricity, that they are ductile and malleable and that they exhibit a nice luster. All these properties derive mainly from the fact that electrons are free to move inside the metal. It is said that their electrons are "delocalized".

We could imagine a metal being formed in two ways:

a) By starting with one atom and adding more and more atoms. In this case the first orbitals that come in contact will split into a bonding and an anti-bonding orbital. These new states will accommodate the valence electrons (those electrons that are in the outside shell) and will not fill the available energy levels. This is crucial, because if all these levels are filled you will end up with either an insulator or a semiconductor. As you add more and more atoms, there will be more and more energy levels, all close to each other. Instead of discrete levels, you can talk about "bands" of energy levels.

b) Start with a lattice of positive ions and add electrons. In this approach, you can think of the lattice as a potential well (recall the particle in a box problem).

Drude Model

Drude proposed in 1900 that the electrons in a metal behave like an ideal gas. He was able to determine an expression for the electrical conductivity:

$$\sigma = \frac{ne^2\tau}{m}$$

Where n is the density of the electrons (in number per unit volume), e and m are the charge and mass of the electron, and τ is the mean time between electron-ion collisions, which could be determined experimentally. This concept of the conductivity limited by collisions is sometimes called the "diffusion theory of electric transport". It is a useful concept even today when you only need a general understanding of how metals conduct electricity, in the same way that we normally use the Rutherford model to describe an atom even though we know it is not completely correct. The Drude model is wrong for several reasons, among them:

- It neglects the electromagnetic interaction between electrons. Electrons are charged particles and they possess spins, but the strong coulomb repulsion is not taken into account! Add to that the fact that the density of electrons in metals is about three orders of magnitude higher than the density of particles in an ideal gas at normal conditions. However, in a very surprising twist of events this is not the most important problem with the model! Even today, the best theoretical models sometimes neglect the e-e interaction and get away with that. On the other hand, there is an area of condensed matter physics that studies specifically the effects of the e-e interaction in situations where it is important. These are called "strongly interacting electrons", as if the interaction were not the same in all cases.

- The Drude model assumes that the distribution of speeds in the metal is the same as in an ideal gas. One consequence of this is that the heat capacity due to the kinetic energy of electrons should be the same as that of an ideal gas ($3/2k_B$ per electron) but in reality it is much less than that. So, what is wrong? Nowadays we know that the true distribution of electron speeds and energies in a metal is given by a very different function that we are going to study next.

Fermi-Dirac Distribution

The distribution of electrons in a metal can be described in terms of the following function:

$$n(E) = g(E)\frac{1}{1+e^{\frac{E-E_F}{k_BT}}}$$

One way to interpret this expression is to say that if you multiply this function by an interval of energy ΔE you get the average number of electrons with energy in the range $[E-1/2\Delta E, E+1/2\Delta E]$, so if the product $n(E)\Delta E$ is 3 for example, it means that there are in average 3 electrons in that energy interval.

In the expression above, g(E) is the density of states, it is a measure of how many levels per joule are available for electrons in the metal. E is the kinetic energy and E_F is called the "Fermi energy".

The factor that multiplies the density of states is called the Fermi-Dirac distribution:

$$F_{FD} = \frac{1}{1+e^{\frac{E-E_F}{k_BT}}}$$

Notice that when T is zero this function is equal to 1 for $E<E_F$ and 0 for $E>E_F$.
So, when T=0 all levels up to the Fermi Energy will be occupied and all levels beyond will be empty.