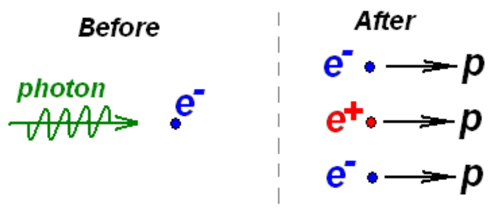
Modern Physics

Pair production

A photon creates an electron and a positron upon colliding with an electron.

If we want the minimum energy to create the pair, we can assume that the three particles have the same momentum and energy after the collision:



Before the collision, the energy is due to the photon and the rest energy of the electron:

$$Total\ Energy = E_{photon} + mc^2$$

Where m is the mass of the electron.

Conservation of energy tells us that:

$E_{photon} + mc^2 = 3E_{electron\ after\ collision}$, so: $E_{electron\ after\ collision} = \dfrac{E_{photon} + mc^2}{3}$

Notice that before the collision the only momentum is from the photon: $p_{photon} = \dfrac{E_{photon}}{c}$

Conservation of linear momentum tells us that: $p_{photon} = 3p_{electron}$

So: $\dfrac{E_{photon}}{c} = \dfrac{3\sqrt{\left(\dfrac{E_{photon} + mc^2}{3}\right)^2 - m^2c^4}}{c}$

Now, solving for the energy of the photon we get:

$$E_{photon} = 3\sqrt{\left(\frac{E_{photon} + mc^2}{3}\right)^2 - m^2c^4} \rightarrow E^2{}_{photon} = 9\left(\frac{E_{photon} + mc^2}{3}\right)^2 - 9m^2c^4$$

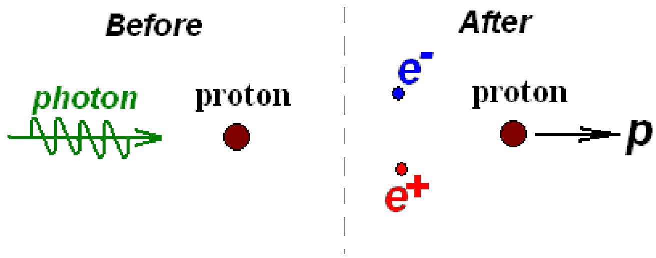
$$E^2{}_{photon} = E^2{}_{photon} + 2E_{photon}mc^2 + m^2c^4 - 9m^2c^4$$

$$\rightarrow 8m^2c^4 = 2E_{photon}mc^2 \rightarrow \boxed{E_{photon} = 4mc^2}$$

So, conservation of linear momentum forces the photon to have a minimum energy of 4 times the rest energy of the electron. That means twice the rest energy of the newly created pair.

A photon creates an electron and a positron upon colliding with a proton.

If we want the minimum energy, we can assume that the proton will take away the momentum of the incident photon and the electron-positron pair is created with zero linear momentum. This is a very close approximation considering how massive the proton is compared to the electron-positron pair:



Conservation of linear momentum tells us that: $\frac{E_{photon}}{c} = p_{proton}$

And conservation of energy tells us that:

$$hf + m_{proton}c^2 = \sqrt{p_{proton}^2 c^2 + m_{proton}^2 c^4} + 2mc^2 ,$$

where m is the mass of the electron (or positron).

Replacing the first equation into the second:

$$E_{photon} + m_{proton}c^2 = \sqrt{E^2{}_{photon} + m_{proton}^2 c^4} + 2mc^2$$

Which we can solve for E_{photon} as follows:

$$E_{photon} + m_{proton}c^2 - 2mc^2 = \sqrt{E^2{}_{photon} + m_{proton}^2 c^4}$$

$$4m^2c^4 + 2E_{photon}m_{proton}c^2 - 4E_{photon}mc^2 - 4m_{proton}mc^4 = 0$$

$$E_{photon} = 2mc^2\left(\frac{m_{proton} - m}{m_{proton} - 2m}\right)$$

With the values:
Mass of the electron $= 9.1\times10^{-31}$ kg
Mass of the proton $= 1.67\times10^{-27}$ kg
The correction in brackets is: $\left(\frac{m_{proton} - m}{m_{proton} - 2m}\right) = 1.000546$

So, you need only a bit more than the energy to create the pair at rest.

Note: The proton taking all the momentum of the photon is an approximation. If we wanted to distribute the momentum among the three particles and minimize the energy we would need less energy, but the correction is in the order of 10^{-8}