# Modern Physics

## Statistical Physics

It is one of the best-developed areas in theoretical physics. It has been applied to problems as simple as the distribution of speeds of an ideal gas and as complicated as the entropy of a black hole.

The most important law of statistical physics is that systems that are in thermal equilibrium are found in a state "i" with a probability that is proportional to the Boltzmann factor of that state:

$$P_i \propto e^{-\frac{E_i}{k_B T}}$$

Where $E_i$ is the energy, T is the absolute temperature and $k_B$ is a constant of nature called "Boltzmann constant" equal to $1.38\times10^{-23}$ J/K. This is not a new law of nature, but rather a consequence of the known laws of probabilities applied to microscopic energy systems ("micro-canonical ensembles" in the jargon of the trade) that are generalized to include systems that exchange heat (canonical ensembles).

We can convert the previous proportionality relation to an equation by realizing that the sum of all probabilities has to be equal to 1. So:

$$P_i = \frac{e^{-\frac{E_i}{k_B T}}}{\sum_j e^{-\frac{E_j}{k_B T}}}$$

The sum in the denominator is a very important function of the absolute temperature, knowing that function allows you to find the entropy, heat capacity and other thermodynamic quantities. It is so important that it is given the name "Partition Function" and sometimes it is represented by the letter "Q" or "Z", although this latter symbol is usually reserved for partition functions of grand-canonical ensembles (systems that exchange not only heat, but particles too).

It is very common that several different states have the same energy. If that is the case, people say it is a degenerate state and then the partition function is given by:

$$Q = \sum_i g_i e^{-\frac{E_i}{k_B T}}$$

Where $g_i$ is the degeneracy of state i.

**Maxwell Velocity Distribution**

$$f(\vec{v}) = \left(\frac{m}{2\pi k_B T}\right)^{3/2} e^{-\frac{1/2 mv^2}{k_B T}}$$

This function describes the distribution of velocities of an ideal gas. It is a classical equation in the sense that the energy is not quantized, but continuous. This function returns a value for a given velocity ($\vec{v}$ goes in, f comes out) whose units are inverse of velocity cubed ($s^3/m^3$). To get a probability you have to multiply the value of f by a volume in phase space, which is a volume in a ($v_x$, $v_y$, $v_z$) space.

Notice that the maximum value of f is at the origin.

**Maxwell Speed Distribution**

$$F(v) = 4\pi v^2 \left(\frac{m}{2\pi k_B T}\right)^{3/2} e^{-\frac{1/2 mv^2}{k_B T}}$$

This function is similar to the velocity distribution, but this time we are not concerned about the direction of the velocity vector. The function describes the probability of finding a particle with *speed* equal to v. The units are inverse of velocity (s/m) so you need to multiply by a velocity interval in m/s to get the probability.

Notice that the maximum of this function is not at the origin, but at a finite value that we can call $v^*$ given by:

$$v^* = \sqrt{\frac{2k_B T}{m}}$$

$v^*$ is the most probable speed of an ideal gas, but the average speed, which is important to determine the average momentum is different:

$$\langle v \rangle = \sqrt{\frac{8}{\pi}}\sqrt{\frac{k_B T}{m}}$$

The average square of the speed is not equal to the square of the average speed but instead it is given by:

$$\langle v^2 \rangle = \frac{3k_B T}{m}$$

The square root of this last quantity is sometimes called "root mean square" and is represented by $v_{rms}$. It is important in determining the average linear kinetic energy of the gas:

$$\left\langle Linear\ Kinetic\ Energy \right\rangle = \frac{1}{2}m\left\langle v^2 \right\rangle = \frac{1}{2}mv_{rms}^2 = \frac{3k_BT}{2}$$