# Modern Physics

## Wien's law

The black body radiation per unit area per unit wavelength is given by the equation:

$$R = \frac{2\pi hc^2}{\lambda^5} \frac{1}{e^{\frac{hc}{\lambda k_B T}} - 1}$$

If we are interested in finding the value of $\lambda_{\max}$, which makes this function a maximum, we can use the usual procedure of taking its derivative and equaling it to zero, as follows.

$$\frac{dR}{d\lambda} = -5\frac{2\pi hc^2}{\lambda^6} \frac{1}{e^{\frac{hc}{\lambda k_B T}} - 1} + \frac{2\pi hc^2}{\lambda^5} \frac{\frac{hc}{\lambda^2 k_B T} e^{\frac{hc}{\lambda k_B T}}}{\left(e^{\frac{hc}{\lambda k_B T}} - 1\right)^2} = 0$$

$$5e^{\frac{hc}{\lambda k_B T}} - 5 = \frac{hc}{\lambda k_B T} e^{\frac{hc}{\lambda k_B T}}$$

To solve this equation, we could assume that the exponential is much larger than 1. With this approximation we find that

$$\lambda_{\max} \approx \frac{hc}{5k_B T} = \frac{2.9\times 10^{-3}\,\mathrm{mK}}{T}$$

A more accurate solution can be found by solving the equation numerically. We change variables to $x = \frac{hc}{\lambda k_B T}$ and the equation becomes $5e^x - 5 = xe^x$, which gives $x = 4.965$, then we get

$$\lambda_{\max} \approx \frac{hc}{4.965 k_B T} = \frac{2.898\times 10^{-3}\,\mathrm{mK}}{T}$$