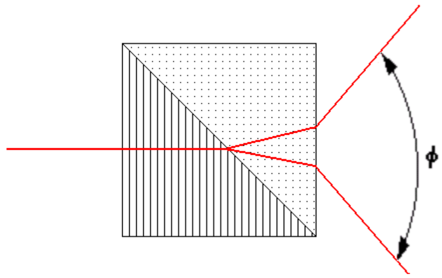
Optics

Birefringence

Problem 1.- Calculate the angle of divergence ϕ of the polarized beams after traversing the arrangement shown in the figure:



Consider the material to have $n_e = 1.58$ and $n_o = 1.62$ with the optical axis rotated 90º in the second prism. Recall that the extraordinary ray becomes ordinary at the interface and vice versa.

Solution: Each polarization will be deviated differently when getting to the interface:

i) Going from n=1.58 to n=1.62

Since the index of refraction in the second crystal is larger, the angle of refraction will be smaller (the beam approaches the normal). Using Snell's law we get:

$$1.58\sin 45^\circ = 1.62\sin\theta_r \rightarrow \theta_r = 43.60^\circ$$

This means that the angle of incidence at the crystal-air interface will be:

$$\theta_i = 45^\circ - 43.60^\circ = 1.40^\circ$$

Using Snell's law again we get the angle of refraction in air:

$$1.62\sin 1.49 = \sin\theta_r \rightarrow \theta_r = 2.26^\circ$$

ii) Going from n=1.62 to n=1.58

Since the index of refraction in the second crystal is now smaller, the angle of refraction will be larger (the beam deviates away from the normal). Using Snell's law we get:

$$1.62\sin 45^\circ = 1.58\sin\theta_r \rightarrow \theta_r = 46.47^\circ$$

This means that the angle of incidence at the crystal-air interface will be:

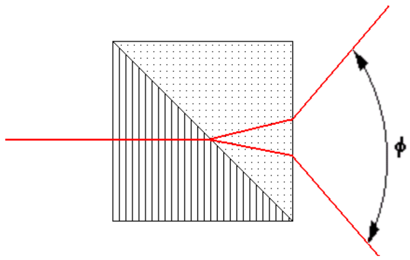
$\theta_i = 46.47^\circ - 45^\circ = 1.47^\circ$ but in the opposite direction as the previous beam

Using Snell's law again we get the angle of refraction in air:

$1.58\sin 1.47 = \sin\theta_r \rightarrow \theta_r = 2.32^{\circ}$

Then the angle between the two beams is $\phi = \mathbf{4.6^{\circ}}$

Problem 2.- Calculate the angle of divergence ϕ of the polarized beams after traversing the arrangement shown in the figure:



Consider the material to have $n_e = 1.48$ and $n_o = 1.52$ with the optical axis rotated 90º in the second prism. As discussed in class, the extraordinary ray becomes ordinary at the interface and vice versa.

Calculate the fraction of intensity of the initial beam that emerges in each direction.

Solution: The extraordinary ray becomes ordinary at the interface and vice versa.
Each polarization will be deviated differently when getting to the interface:

i) Going from n=1.48 to n=1.52

Since the index of refraction in the second crystal is larger, the angle of refraction will be smaller (the beam approaches the normal). Using Snell's law we get:

$1.48\sin 45^{\circ} = 1.52\sin\theta_r \rightarrow \theta_r = 43.51^{\circ}$

This means that the angle of incidence at the crystal-air interface will be:

$\theta_i = 45^{\circ} - 43.51^{\circ} = 1.49^{\circ}$

Using Snell's law again we get the angle of refraction in air:

$1.52\sin 1.49 = \sin\theta_r \rightarrow \theta_r = 2.26^{\circ}$

ii) Going from n=1.658 to n=1.486

Since the index of refraction in the second crystal is now smaller, the angle of refraction will be larger (the beam deviates away from the normal). Using Snell's law we get:

$1.52\sin 45^{\circ} = 1.48\sin\theta_r \rightarrow \theta_r = 46.57^{\circ}$
This means that the angle of incidence at the crystal-air interface will be:

$\theta_i = 46.57^\circ - 45^\circ = 1.57^\circ$ but in the opposite direction as the previous beam.

Using Snell's law again we get the angle of refraction in air:

$1.48 \sin 1.57 = \sin\theta_r \rightarrow \theta_r =$ 2.32º

Then the angle between the two beams is $\phi = \mathbf{4.6^o}$

The fraction of intensity of the initial beam that emerges in each direction:

Let us analyze the TE and TM modes separately

TE mode: There are three interfaces

i) Air-crystal interface, $n_i = 1, n_t = 1.48$, $\theta_i = 0$ and $\dfrac{E_t}{E_i} = 0.8064$

ii) Crystal-crystal interface, $n_i = 1.48, n_t = 1.52$, $\theta_i = 45$ and $\dfrac{E_t}{E_i} = 0.9740$

iii) Crystal-air interface, $n_i = 1.52, n_t = 1$, $\theta_i = 1.49$ and $\dfrac{E_t}{E_i} = 1.206$

To find the ratio between the initial intensity and the one emerging from the splitter notice that the intensity is proportional to the electric field square and we can assume that 50% of the original intensity was polarized in the TE mode, then:

$$\frac{I_{TE-emerging}}{I_o} = 0.5 \times 0.8064^2 \times 0.9740^2 \times 1.206^2 = 0.449$$

TM mode: There are also three interfaces

iv) Air-crystal interface, $n_i = 1, n_t = 1.52$, $\theta_i = 0$ and $\dfrac{E_t}{E_i} = 0.7937$

v) Crystal-crystal interface, $n_i = 1.52, n_t = 1.48$, $\theta_i = 45$ and $\dfrac{E_t}{E_i} = 1.028$

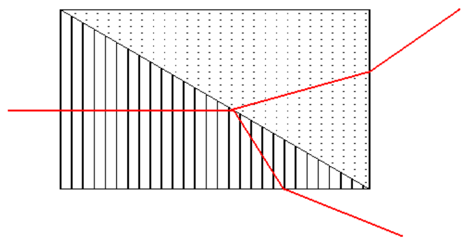
vi) Crystal-air interface, $n_i = 1.48, n_t = 1$, $\theta_i = 1.57$ and $\dfrac{E_t}{E_i} = 1.194$

Similar to the TE mode, to find the ratio between the initial intensity and the one emerging from the splitter notice that the intensity is proportional to the electric field squared and we can assume that 50% of the original intensity was polarized in the TM mode, so:

$$\frac{I_{TM-emerging}}{I_o} = 0.5 \times 0.7937^2 \times 1.028^2 \times 1.194^2 = 0.474$$

So, we have a slightly more intense TM mode.

Problem 3.- Calculate the angle of incidence necessary to totally reflect one polarization and leave the other one when natural light goes through the arrangement shown below. A crystal of calcite is shown with two regions that have the optical axis rotated 90º



Solution: To get total internal reflection the index of refraction should go from 1.6584 to 1.4864 and the critical angle will be:

$$\theta_c = \sin^{-1}\left(\frac{1.4864}{1.6584}\right) = \mathbf{63.7^o}$$

An angle larger than that will guarantee total internal reflection for that polarization.