Optics

Diffraction limit

Problem 1.- When light goes through a circular aperture the diffraction pattern produced is similar to the one generated by a single slit, but circular. The central maximum goes to zero at an angle $\theta = \sin^{-1}\left(1.22\frac{\lambda}{D}\right)$, where D is the diameter of the aperture. This angle is considered the ultimate limit in resolution imposed by diffraction.
Consider a camera mounted on a satellite 300km above the ground. Calculate the minimum size of the objective to distinguish two objects separated 0.45m.
Consider $\lambda = 600\text{nm}$

Solution: The separation between the objects divided by the height will give us the angle (in radians):
$\theta = \frac{0.45m}{300\times10^{3}m} = 1.5\times10^{-6}\,radians$, but we also know $\theta = \sin^{-1}\left(1.22\frac{\lambda}{D}\right)$, so the diameter will be:

$$1.5\times10^{-6} = \sin^{-1}\left(1.22\frac{600\times10^{-9}}{D}\right) \rightarrow D = \mathbf{0.49\ m}$$

Problem 2.- Two stars are separated by an angle of 3×10^{-6} radians. What is the diameter of the smallest telescope that can distinguish the two stars using visible light (wavelength 600 nanometers)?

(A) 2.5 mm
(B) 2.5 cm
(C) 25 cm
(D) 2.5 m
(E) 25 m

Solution: The diffraction limit of resolution is: $\theta = \sin^{-1}\left(\frac{1.22\lambda}{D}\right)$, so the diameter will be:

$$D = \frac{1.22\lambda}{\sin\theta} = \frac{1.22\times600\times10^{-9}}{3\times10^{-6}} = 0.244\text{ m}$$

Answer **(C) 25 cm**

Problem 3.- Light from a laser falls on a pair of very narrow slits separated by 0.5 micrometers, and bright fringes separated by 1.0 millimeter are observed on a distant screen. If the wavelength of the laser light is doubled, what will be the separation of the bright fringes?

(A) 0.25 mm
(B) 0.5 mm
(C) 1.0 mm
(D) 2.0 mm
(E) 2.5 mm

Solution: If the wavelength of the laser light is doubled, the angle given by $\theta = \sin^{-1}\left(\frac{\lambda}{D}\right)$ will increase too, so answer: **(D) 2.0 mm**

Problem 4.- One of the earliest diffraction gratings was made with thin wires separated by just $5\,\mu\text{m}$. Calculate the angles that correspond to the first diffraction of the visible spectrum (take the limits as 400nm and 700nm)

Solution:

$$\theta_{400\text{nm}} = \sin^{-1}\left(\frac{\lambda}{d}\right) = \sin^{-1}\left(\frac{400\times10^{-9}}{5\times10^{-6}}\right) = \mathbf{4.58^{o}}$$

$$\theta_{700\text{nm}} = \sin^{-1}\left(\frac{\lambda}{d}\right) = \sin^{-1}\left(\frac{700\times10^{-9}}{5\times10^{-6}}\right) = \mathbf{8.04^{o}}$$

Problem 5.- A carbon nanotube of only 20nm in diameter is illuminated with soft X-rays from a coherent source of wavelength 1nm. Calculate the angle corresponding to the first dark fringe. According to Babinet's theorem the diffraction pattern due to an obstacle is the same as the one of an aperture of the same shape.

Solution:

$$\theta = \sin^{-1}\left(\frac{\lambda}{W}\right) = \sin^{-1}\left(\frac{1\times10^{-9}}{20\times10^{-9}}\right) = \mathbf{2.87^{o}}$$