Optics

EM waves

Problem 1.- Describe the following electromagnetic waves in terms of direction of propagation and polarization:
[The functions given are the electric field vectors]

a) $\vec{E} = E_o \hat{z} \cos(\omega t + ky)$

b) $\vec{E} = \hat{x} \cos(\omega t - kz) + \hat{y} \cos(\omega t - kz)$

c) $\vec{E} = \hat{y} E_o \cos(\omega t) \cos(kx)$

Solution:
a) $\vec{E} = E_o \hat{z} \cos(\omega t + ky)$
It's a wave propagating in the negative y direction and its plane polarized.

b) $\vec{E} = \hat{x} \cos(\omega t - kz) + \hat{y} \cos(\omega t - kz)$
It's a wave propagating in the positive z direction and its plane polarized.

c) $\vec{E} = \hat{y} E_o \cos(\omega t) \cos(kx)$
Two waves propagating in the positive and negative x direction plane polarized.

Problem 2.- Consider the wave described by:
$\vec{E} = 3\hat{x} \cos(\omega t - kz) + 3\hat{y} \cos(\omega t - kz)$
Indicate the state of polarization after going through a 1/2 wave plate with the fast axis in the x direction.

Solution: After going through the 1/2 wave plate the new field will be:

$$\vec{E} = 3\hat{x} \cos(\omega t - kz + \pi) + 3\hat{y} \cos(\omega t - kz) = -3\hat{x} \cos(\omega t - kz) + 3\hat{y} \cos(\omega t - kz)$$

So it is still a plane polarized wave, but the x-component of the field changed sign, so this is equivalent to a rotation of 90º.

Problem 3.- Which of the following equations corresponds to the electric field of a plane polarized electromagnetic wave moving in the negative z direction?

(A) $\vec{E} = E_o \hat{x} \cos(\omega t - kz) + E_o \hat{y} \cos(\omega t - kz)$

(B) $\vec{E} = E_o \hat{x} \cos(\omega t - kz) + E_o \hat{y} \sin(\omega t - kz)$

(C) $\vec{E} = E_o \hat{x} \cos(\omega t + kz) + E_o \hat{y} \cos(\omega t + kz)$

(D) $\vec{E} = E_o \hat{x} \cos(\omega t + kz) + E_o \hat{y} \sin(\omega t + kz)$

(E) $\vec{E} = E_o \hat{x} \cos(\omega t + kz) + E_o \hat{y} \cos(\omega t + kz - \pi / 2)$

Solution:
We identify **C**

Problem 4.- An electromagnetic wave in vacuum has an electric field described by the equation $\vec{E} = E_o \hat{x} \cos(\omega t - kz)$

Indicate:
Direction of propagation of the wave:
Direction of the magnetic field:
What kind of polarization it is:

Solution:
Direction of propagation of the wave: **Positive z-axis**
Direction of the magnetic field: **y-axis**
What kind of polarization it is: **Plane**

Problem 5.- Identify the direction of propagation of the following waves and what kind of polarization they represent.

a) $\vec{E} = \hat{x}E_o \cos(\omega t + ky) + E_o \hat{z} \cos(\omega t + ky)$

b) $\vec{E} = \hat{x}E_o \cos(\omega t - kz) + \hat{y}\left(\frac{E_o}{2}\right)\cos(\omega t - kz)$

c) $\vec{E} = \hat{x}E_o \cos(\omega t - ky) + \hat{z}E_o \sin(\omega t - ky)$

d) $\vec{E} = \hat{x}E_o \cos(\omega t)\cos(kz)$

c) $\vec{E} = \hat{x}E_o \cos\left(\omega t - \frac{ky}{\sqrt{2}} + \frac{kz}{\sqrt{2}}\right)$

Solution:
a) $\vec{E} = \hat{x}E_o \cos(\omega t + ky) + E_o \hat{z} \cos(\omega t + ky)$
This wave travels in the negative y direction and it is plane polarized (*P*)

b) $\vec{E} = \hat{x}E_o \cos(\omega t - kz) + \hat{y}\left(\frac{E_o}{2}\right)\cos(\omega t - kz)$
This wave travels in the positive z direction and it is plane polarized (*P*)

c) $\vec{E} = \hat{x}E_o \cos(\omega t - ky) + \hat{z}E_o \sin(\omega t - ky)$
This wave travels in the positive y direction and it is circularly polarized (*P*)

d) $\vec{E} = \hat{x}E_o \cos(\omega t)\cos(kz)$

Notice that: $\vec{E} = \hat{x}E_o \cos(\omega t)\cos(kz) = \hat{x}\frac{E_o}{2}\left[\cos(\omega t + kz) + \cos(\omega t - kz)\right]$

This is a standing wave, which can be represented by two plane polarized waves traveling in opposite directions along the z axis.

c) $\vec{E} = \hat{x}E_o \cos\left(\omega t - \frac{ky}{\sqrt{2}} + \frac{kz}{\sqrt{2}}\right)$

The polarization is linear because the electric field vector always points in the x-direction.
The wave travels in a direction that bisects the angle between the positive y and negative z directions.

Problem 6.- Consider the wave described by:

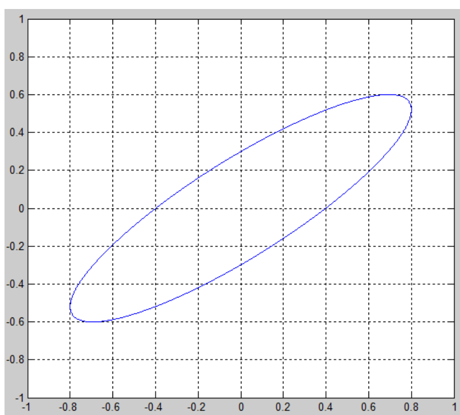
$$\vec{E} = \hat{x}\left(\frac{4}{5}\right)\cos(\omega t - kz) + \hat{y}\left(\frac{3}{5}\right)\cos(\omega t - kz - 30^\circ)$$

Show with a graph how the electric field oscillates in the x-y plane.
Identify from the graph (or mathematically) the angle between the major axis of the ellipse and the x-axis.
[Suggestion: instead of manually graphing the vectors on graph paper, use a spreadsheet program to do it for you and print the graph]

Solution: The electric field wave described by:

$$\vec{E} = \hat{x}\left(\frac{4}{5}\right)\cos(\omega t - kz) + \hat{y}\left(\frac{3}{5}\right)\cos(\omega t - kz - 30^\circ)$$

is shown in the graph below:



It is a case of elliptical polarization, with the ellipse rotated at an angle of 35.7º

This can also be calculated mathematically instead of graphically. We can use the equation:

$$\alpha = \frac{1}{2}\tan^{-1}\left(\frac{2E_x E_y \cos\phi}{E_x^2 - E_y^2}\right) = \frac{1}{2}\tan^{-1}\left(\frac{2(4/5)(3/5)\cos 30^\circ}{(4/5)^2 - (3/5)^2}\right) = 35.7^\circ$$

Problem 7.- Consider the wave described by:

$$\vec{E} = \hat{x}\cos(\omega t - kz) + 2\hat{y}\cos(\omega t - kz - \pi/2)$$

Show with graph how the electric field oscillates in the x-y plane (z=0). Indicate the direction of oscillation (clockwise or anticlockwise).

Solution: In the graph below anticlockwise rotation of the electric field: