Optics

Fresnel equations

Problem 1.- Suppose a beam of light is polarized in the plane of incidence (the electric field is parallel to the plane of incidence) and impinges on a diamond surface at an angle of 45º. Calculate the reflection and transmission coefficients.
Repeat the calculations for a beam that is polarized perpendicularly to the plane of incidence (the electric field is perpendicular to the plane of incidence).

Solution: To calculate the reflection and transmission coefficients we need the angle of refraction, so we apply Snell's law at the interface air-diamond:

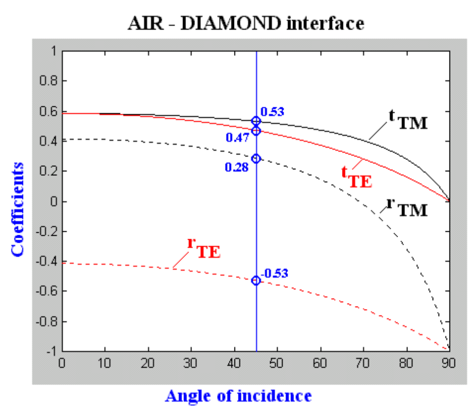
$$n_i \sin\theta_i = n_t \sin\theta_t \rightarrow 1\times\sin 45^\circ = 2.42\times\sin\theta_t \rightarrow \theta_t = \sin^{-1}\left(\frac{\sin 45^\circ}{2.42}\right) = 17º$$

TM Mode:
$$\frac{E_r}{E_i} = \frac{n_t\cos\theta_i - n_i\cos\theta_t}{n_t\cos\theta_i + n_i\cos\theta_t} = \frac{2.42\times\cos 45^\circ - 1\times\cos 17^\circ}{2.42\times\cos 45^\circ + 1\times\cos 17^\circ} = \mathbf{0.28}$$

and
$$\frac{E_t}{E_i} = \frac{2n_i\cos\theta_i}{n_t\cos\theta_i + n_i\cos\theta_t} = \frac{2\times 1\times\cos 45^\circ}{2.42\times\cos 45^\circ + 1\times\cos 17^\circ} = \mathbf{0.53}$$

TE Mode:
$$\frac{E_r}{E_i} = \frac{n_i\cos\theta_i - n_t\cos\theta_t}{n_i\cos\theta_i + n_t\cos\theta_t} = \frac{1\times\cos 45^\circ - 2.42\times\cos 17^\circ}{1\times\cos 45^\circ + 2.42\times\cos 17^\circ} = \mathbf{-0.53}$$

and
$$\frac{E_t}{E_i} = \frac{2n_i\cos\theta_i}{n_i\cos\theta_i + n_t\cos\theta_t} = \frac{2\times 1\times\cos 45^\circ}{1\times\cos 45^\circ + 2.42\times\cos 17^\circ} = \mathbf{0.47}$$



Problem 2.- Suppose you have to illuminate a sample inside a vacuum chamber through a glass window of index of refraction n=1.52 at an angle of incidence of 33º. Find the percentage of intensity lost due to reflection at the first air-glass interface in the TE mode.

Solution: According to Snell's law, the transmission angle is:

$$\sin 33^\circ = 1.52 \sin\theta_t \to \theta_t = \sin^{-1}\left(\frac{\sin 33^\circ}{1.52}\right) = 21^\circ$$

Now we can use Fresnel equations to find the ratio between E_r and E_i:

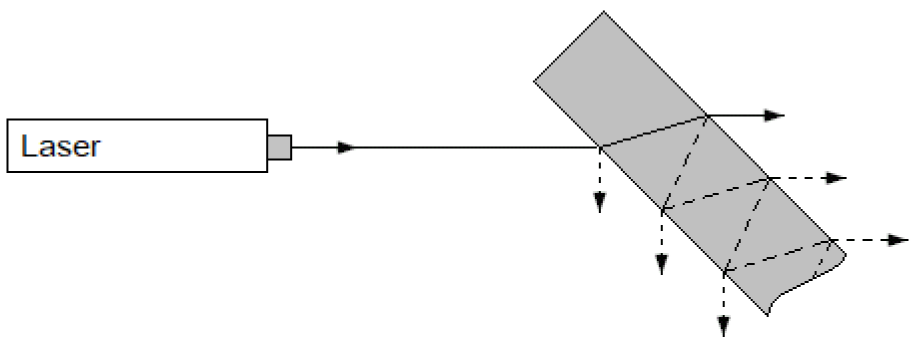
$$\frac{E_r}{E_i} = \frac{n_i \cos\theta_i - n_t \cos\theta_t}{n_i \cos\theta_i + n_t \cos\theta_t} = \frac{\cos 33^\circ - 1.52\cos 21^\circ}{\cos 33^\circ + 1.52\cos 21^\circ} = -0.257$$

To get the fraction of the intensity reflected we square this ratio:

$$\left(\frac{E_r}{E_i}\right)^2 = (-0.257)^2 = 0.066$$

So **6.6%** is lost.

Problem 3.- For a beam of light polarized in the plane of incidence calculate the fraction of the electric field that will be transmitted to the other side if the incident angle is 45º as shown in the figure. Consider only the beam that is shifted once. The index of refraction of the glass is 1.52



Solution: A beam of light polarized in the plane of incidence is called the TM mode and we need to perform the calculation of the transmission coefficient twice:

At the air-glass interface:

$$n_i \sin\theta_i = n_t \sin\theta_t \to 1\times\sin 45^\circ = 1.52\times\sin\theta_t \to \theta_t = \sin^{-1}\left(\frac{\sin 45^\circ}{1.52}\right) = \mathbf{27.7^o}$$

So the transmission coefficient is:

$$\frac{E_t}{E_i} = \frac{2n_i \cos\theta_i}{n_t \cos\theta_i + n_i \cos\theta_t} = \frac{2\times1\times\cos 45^\circ}{1.52\times\cos 45^o + 1\times\cos 27.7^\circ} = \mathbf{0.721}$$

And at the glass-air interface the transmission coefficient is:

$$\frac{E_t}{E_i} = \frac{2n_i \cos\theta_i}{n_t \cos\theta_i + n_i \cos\theta_t} = \frac{2\times1.52\times\cos 27.7^\circ}{1\times\cos 27.7^o + 1.52\times\cos 45^\circ} = \mathbf{1.373}$$

So the overall fraction is: 0.721×1.373=**0.99**

Problem 4.- For a beam of light polarized perpendicular to the plane of incidence (TE mode) calculate how much light is reflected at an angle of incidence of 30º from a surface of diamond (n=2.42).
[Calculate the ratio of the electric fields reflected/incident]

Solution: To calculate the reflection coefficient we need the angle of refraction, so we apply Snell's law at the interface air-diamond:

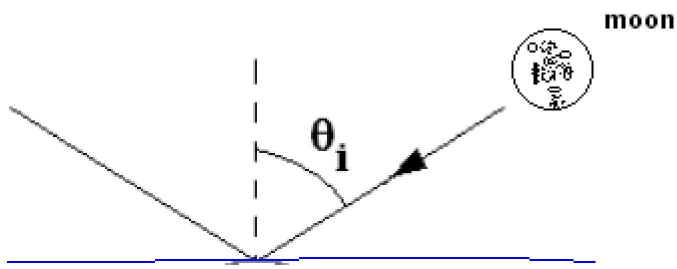
$$n_i \sin\theta_i = n_t \sin\theta_t \rightarrow 1\times\sin 30^\circ = 2.42\times\sin\theta_t \rightarrow \theta_t = \sin^{-1}\left(\frac{\sin 30^\circ}{2.42}\right) = 11.9^\circ$$

$$\frac{E_r}{E_i} = \frac{n_i\cos\theta_i - n_t\cos\theta_t}{n_i\cos\theta_i + n_t\cos\theta_t} = \frac{1\times\cos 30^\circ - 2.42\times\cos 11.9^o}{1\times\cos 30^\circ + 2.42\times\cos 11.9^o} = \mathbf{-0.46}$$

Problem 5.- Suppose you need to design a window that allows all the light polarized in the plane of incidence to get transmitted. At what angle do you need to tilt the window?
Consider that the window has an index of refraction of 1.54

Solution: The angle that we need is Brewster's: $\theta_p = \tan^{-1}(1.54) = \mathbf{57^\circ}$

Problem 6.- Calculate the angle of incidence of moonlight on the surface of a lake, so the reflected light is totally polarized perpendicular to the plane of incidence.
[Take the index of refraction of water = 1.33]



Solution: Once again what we need is Brewster's angle: $\theta_p = \tan^{-1}(1.33) = \mathbf{53^\circ}$