Optics

Geometrical optics

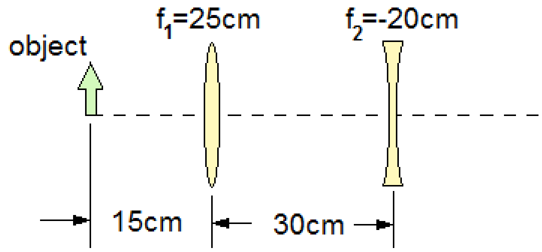
Lens maker's equation $\quad \frac{1}{f} = (n-1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$ with positive radii if convex

Geometric optics equations for lenses and mirrors

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} \qquad m = -\frac{d_i}{d_o} = \frac{h_i}{h_o}$$

For spherical mirrors $f = \pm\frac{R}{2}$, positive if concave

Problem 1.- Calculate where the final image is formed in this combination of lenses.



Solution: The first lens equation is

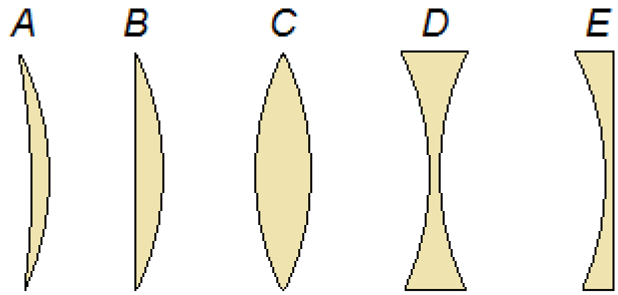
$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} \rightarrow \frac{1}{25} = \frac{1}{15} + \frac{1}{d_i} \rightarrow d_i = \frac{1}{\frac{1}{25} - \frac{1}{15}} = \frac{25 \times 15}{15 - 25} = -37.5$$

This means that the image produced by the first lens will be 37.5cm to the left of the first lens and 67.5cm to the left of the second lens. Now we can use this image as the object for the second lens

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} \rightarrow \frac{1}{-20} = \frac{1}{67.5} + \frac{1}{d_i} \rightarrow d_i = \frac{1}{-\frac{1}{20} - \frac{1}{67.5}} = -\frac{67.5 \times 20}{20 + 67.5} = \mathbf{-15.4\ cm}$$

So the image is formed 15.4 cm to the left of the second lens.

Problem 2.- If the five lenses shown below are made of glass (n=1.5), which lens has the longest positive focal length?



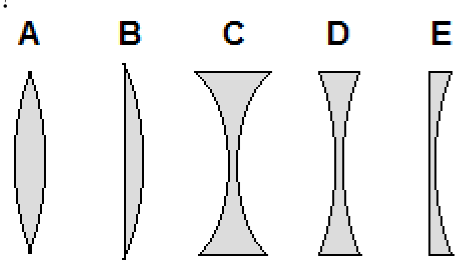
Solution: Notice that D and E are negative lenses, so the choice is between A, B and C.
Also notice that between B and C the focal length of C will be shorter because it has two curved surfaces instead of one.
So the final choice for longest focal length is between A and B. Let us look at the lens maker's equation:

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

In case B the contribution of the flat side is zero, because $\frac{1}{R_1}$ is zero, but in case A the contribution of $\frac{1}{R_1}$ is negative. The focal length is still positive but the value in brackets $\frac{1}{R_1} + \frac{1}{R_2}$ is smaller than $\frac{1}{R_2}$, so the focal length of A is longer.

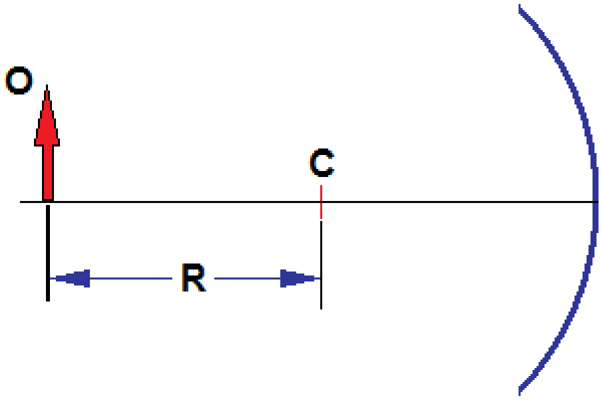
Answer: **A**

Problem 2a.- If the five lenses shown below are made of glass (n=1.5), which lens has the shortest negative focal length?



Problem 3.- The figure below shows an object O placed at a distance R to the left of the center of curvature of a concave spherical mirror that has a radius R. The image formed by the mirror is at

(A) infinity
(B) a distance 2R/3 to the left of the mirror and inverted
(C) a distance 2R/3 to the right of the mirror and upright
(D) a distance 2R/5 to the left of the mirror and inverted
(E) a distance 2R/5 to the right of the mirror and upright

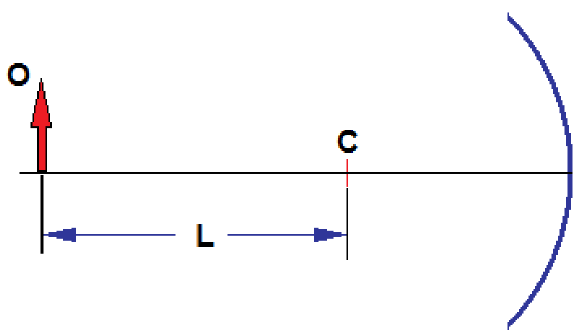


Solution:
Using the fundamental equation of geometrical optics:

$$\frac{1}{f}=\frac{1}{d_o}+\frac{1}{d_i}\rightarrow\frac{2}{R}=\frac{1}{2R}+\frac{1}{d_i}\rightarrow\frac{3}{2R}=\frac{1}{d_i}\rightarrow d_i=\frac{2R}{3}$$

Answer: **B**

Problem 3a.- The figure shows a spherical concave mirror with center of curvature at C and radius R. An object is located a distance L=1.5R from the center as shown in the figure. Calculate the position of the image formed by the mirror.

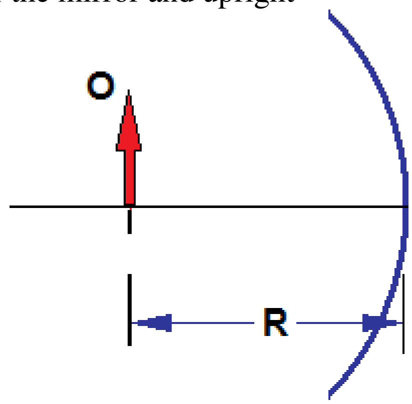


Solution: The focal length of the mirror is R/2, positive and according to the figure the value of d_o is 2.5R, so the equation is

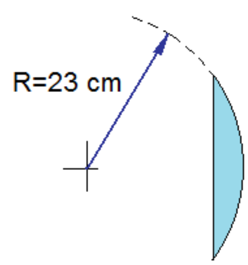
$$\frac{1}{f}=\frac{1}{d_o}+\frac{1}{d_i}\rightarrow\frac{1}{R/2}=\frac{1}{2.5R}+\frac{1}{d_i}\rightarrow d_i=\frac{1}{\frac{1}{R/2}-\frac{1}{2.5R}}=\frac{R}{2-0.4}=\frac{R}{1.6}=\mathbf{0.625\ R}$$

Problem 3b.- The figure below shows an object O placed at a distance R to the left of a concave spherical mirror that has a radius R (so the object is at the center of the sphere). The image formed by the mirror is at

(A) infinity
(B) a distance R to the left of the mirror and inverted
(C) a distance R to the right of the mirror and upright
(D) a distance R/2 to the left of the mirror and inverted
(E) a distance R/2 to the right of the mirror and upright



Problem 4.- A survival kit contains a thin plano-convex lens made of glass (n=1.46) with a spherical surface of 23cm radius. Find how far from a bunch of straw you need to hold the lens to start a fire with the sun's rays.

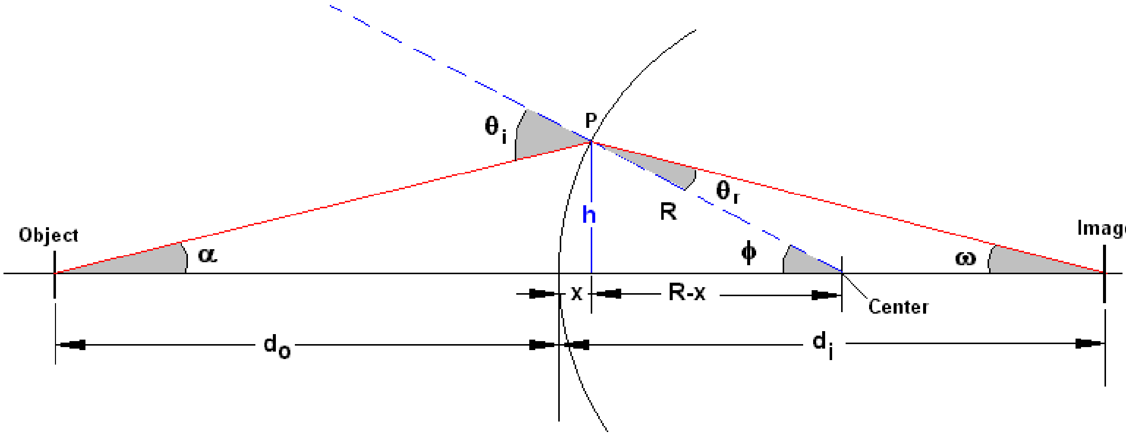


Solution: We can use the "Lens maker equation" to calculate the focal length

$$\frac{1}{f}=\frac{1.46-1}{23} \rightarrow f=\frac{23}{0.46}=\mathbf{50\ cm}$$

That is the answer because if the sun is at infinity, the rays are going to converge at the focal point.

Problem 5.- Considering the geometry shown in the figure, calculate the location of the image created by light striking the spherical transparent surface at point P.
Take d_o= 2.5 cm, R= 1.2 cm, n=2.42, Φ=15°



Solution:

$$h = R\sin\phi = 1.2\sin 15^\circ = 0.3106$$

$$x = R - R\cos\phi = 1.2(1-\cos 15^\circ) = 0.0409$$

$$\alpha = \tan^{-1}\left(\frac{h}{d_o + x}\right) = \tan^{-1}\left(\frac{0.3106}{2.5+0.0409}\right) = 6.969^\circ$$

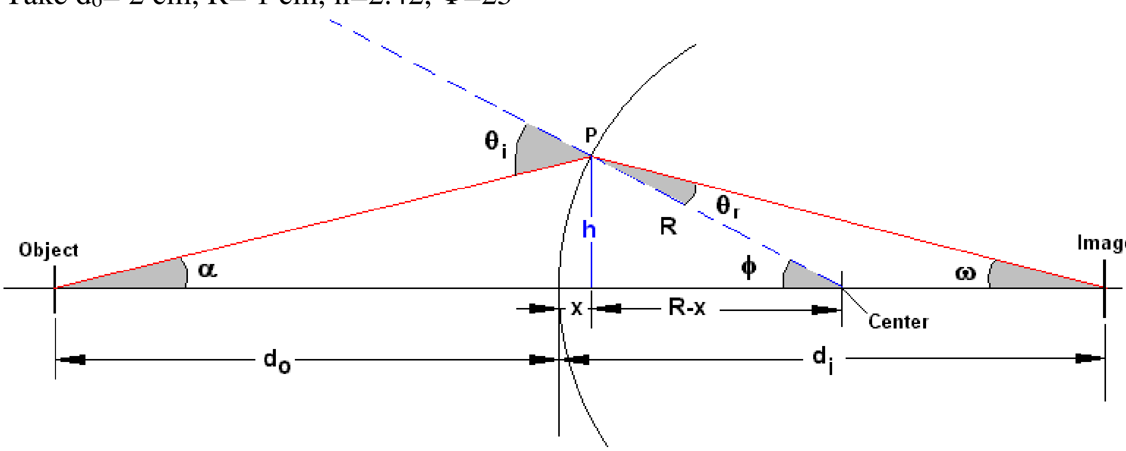
$$\theta_i = \alpha + \phi = 15^\circ + 6.969^\circ = 21.969^\circ$$

$$\sin\theta_i = n\sin\theta_r \rightarrow \theta_r = \sin^{-1}\left(\frac{\sin\theta_i}{n}\right) = \sin^{-1}\left(\frac{\sin 21.969^\circ}{2.42}\right) = 8.89^\circ$$

$$\omega = \phi - \theta_r = 15^\circ - 8.89^\circ = 6.11^\circ$$

$$d_i = \frac{h}{\tan\omega} + x = \frac{0.3106}{\tan 6.11^\circ} + 0.0409 = \mathbf{2.94}$$

Problem 5a.- Considering the geometry shown in the figure, calculate the location of the image created by light striking the spherical transparent surface at point P.
Take d_o= 2 cm, R= 1 cm, n=2.42, Φ=25°



Solution:

$$h = R\sin\phi = \sin 25^\circ = 0.4226$$
$$x = R(1-\cos\phi) = 1-\cos 25^\circ = 0.0937$$
$$\alpha = \tan^{-1}\left(\frac{h}{d_o + x}\right) = \tan^{-1}\left(\frac{0.4226}{2+0.0937}\right) = 11.41^\circ$$
$$\theta_i = \alpha + \phi = 36.41^\circ$$
$$\theta_r = \sin^{-1}\left(\frac{\sin\theta_i}{n}\right) = \sin^{-1}\left(\frac{\sin 36.41^\circ}{2.42}\right) = 14.19^\circ$$
$$\omega = \phi - \theta_r = 10.8^\circ$$
$$d_i = x + \frac{h}{\tan\omega} = 0.0937 + \frac{0.4226}{\tan 14.19^\circ} = \mathbf{2.309\ cm}$$

Problem 6.-

a) A Christmas tree ornament creates an image of your face that is upright and 18 times smaller. What is the radius of the ball if your face is 24cm away?

b) A very large and well polished bowl (but not a super one) shows your face upright and 2.5 times larger. How far away is your face if the spherical shape of the bowl has a radius of 85 cm?

Solution:

a) An upright image means that the magnification is positive and being 18 times smaller means that:

$$M = \frac{h_i}{h_o} = -\frac{d_i}{d_o} = \frac{1}{18}$$

We also know the value of $d_o = 24cm$, so using the equation above:

$$-\frac{d_i}{24} = \frac{1}{18} \to d_i = -\frac{24}{18}cm$$

Now we can use the mirror equation to find the focal length:

$$\frac{1}{d_i} + \frac{1}{d_o} = \frac{1}{f} \to \frac{1}{-(24/18)} + \frac{1}{24} = \frac{1}{f} \to f = -\frac{24}{17}cm = -1.41cm$$

And the radius is twice this value (the sign of *f* means it's convex): R=**2.82cm**

b) In this case the magnification equation is $M = \frac{h_i}{h_o} = -\frac{d_i}{d_o} = 2.5$ so we can write: $d_i = -2.5d_o$

The focal length, which is half the radius, is 42.5cm (positive because it is a concave mirror). So using the mirror equation we get:

$$\frac{1}{d_i} + \frac{1}{d_o} = \frac{1}{f} \to \frac{1}{-2.5d_o} + \frac{1}{d_o} = \frac{1}{42.5} \to \frac{0.6}{d_o} = \frac{1}{42.5} \to d_o = 0.6\times 42.5cm = \mathbf{25.5cm}$$

Problem 7.- A light source is at the bottom of a pool of alcohol (index of refraction is 1.36). At what minimum angle of incidence will a ray be totally reflected at the surface?

(A) 0º
(B) 22º
(C) 27º
(D) 47º
(E) 60º

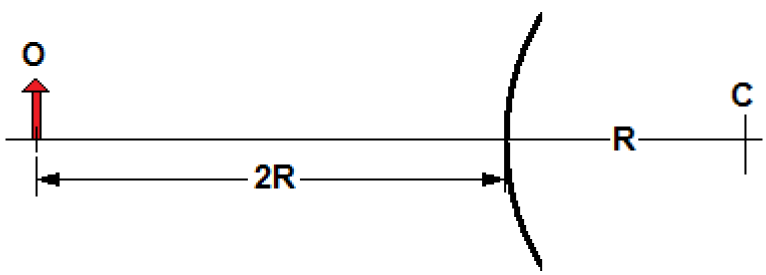
Solution: Finding the critical angle for total internal reflection we get

$$\theta_{critical} = \sin^{-1}\left(\frac{1}{n}\right) = \sin^{-1}\left(\frac{1}{1.36}\right) = 47^\circ$$

Answer **(C) 27º**

Problem 8.- The figure below shows an object O placed at a distance 2R to the left of a convex spherical mirror that has a radius of curvature R. Point C is the center of curvature of the mirror. The image formed by the mirror is at

(A) infinity
(B) a distance 2R to the left of the mirror and inverted
(C) a distance 2R to the right of the mirror and upright
(D) a distance 2R/5 to the left of the mirror and inverted
(E) a distance 2R/5 to the right of the mirror and upright



Problem 9.- Find the radius of a concave polished bowl that shows your face upright and 2.5 times larger when you are 20 cm away.

Solution: In this case the magnification equation is $M = \frac{h_i}{h_o} = -\frac{d_i}{d_o} = 2.5$ and $d_o = 20cm$ so we can write: $d_i = -2.5d_o = -50cm$

Using the mirror equation we get:

$$\frac{1}{d_i} + \frac{1}{d_o} = \frac{1}{f} \rightarrow \frac{1}{-50} + \frac{1}{20} = \frac{1}{f} \rightarrow f = \frac{1}{0.05 - 0.02} = \frac{1}{0.03} \rightarrow f = 33.3 \text{ cm}$$

The radius is twice this number: R = 2f = **66.6 cm**