Optics

Half and quarter wave plate

Problem 1.- Consider a slab of calcite with the optical axis in the vertical direction illuminated with plane polarized light from a sodium lamp ($\lambda = 589\text{nm}$).

a) Calculate the thickness necessary to generate a half wave plate out of this material.

b) Determine the polarization state after going through the plate if the plane of polarization makes and angle of 45° with the vertical axis.

c) Calculate the thickness necessary to generate a quarter wave plate out of this material.

b) Determine the polarization state after going through the plate if the plane of polarization makes and angle of 45° with the vertical axis.

Solution:

a) A half wave plate produces a delay of T/2 in the slower beam.

$$t_{fast} = \frac{\text{thickness}}{c/n_{fast}} = n_{fast}\frac{\text{thickness}}{c}$$

$$t_{slow} = \frac{\text{thickness}}{c/n_{slow}} = n_{slow}\frac{\text{thickness}}{c}$$

$$\rightarrow t_{slow} - t_{fast} = (n_{slow} - n_{fast})\frac{\text{thickness}}{c}$$ this has to be T/2, so:

$$\rightarrow \frac{T}{2} = (n_{slow} - n_{fast})\frac{\text{thickness}}{c} \rightarrow \text{thickness} = \frac{cT}{2(n_{slow} - n_{fast})} = \frac{\lambda}{2(n_{slow} - n_{fast})}$$

Given the values of the problem, we get:

$$\text{thickness} = \frac{589\text{nm}}{2(1.658 - 1.486)} = \mathbf{1.712\ \mu m}$$

Note: You should know that this is not the only thickness that will produce a half wave plate. If the delay is any integer and a half periods, the plate will generate the required delay.

b) Assume the fast axis is the x-axis, the slow axis is the y-axis and the incident beam has electric field components as written below:

$$E_x = E_\circ \cos 45^\circ \cos(kz - \omega t)$$

$$E_y = E_\circ \sin 45^\circ \cos(kz - \omega t)$$

Then after going through the plate, we will have:

$$E_x = E_\circ \cos 45^\circ \cos(kz - \omega t)$$
$$E_y = E_\circ \sin 45^\circ \cos(kz - \omega t - 180^\circ) = -E_\circ \sin 45^\circ \cos(kz - \omega t)$$

Which is a beam plane polarized at 90º with respect to the original beam.

c) A quarter wave plate produces a delay of T/4 in the slower beam.

$$thickness = \frac{cT}{4(n_{slow} - n_{fast})} = \frac{\lambda}{4(n_{slow} - n_{fast})}$$

Given the values of the problem, we get:

$$thickness = \frac{589nm}{4(1.658 - 1.486)} = \mathbf{0.856\ \mu m}$$

Note: This is not the only thickness that will produce a quarter wave plate. If the delay is any integer plus a quarter periods, the plate will generate the required delay.

d) Assume the fast axis is the x-axis, the slow axis is the y-axis and the incident beam has electric field components as written below:

$$E_x = E_\circ \cos 45^\circ \cos(kz - \omega t)$$
$$E_y = E_\circ \sin 45^\circ \cos(kz - \omega t)$$

Then after going through the plate, we will have:

$$E_x = E_\circ \cos 45^\circ \cos(kz - \omega t)$$
$$E_y = E_\circ \sin 45^\circ \cos(kz - \omega t - 90^\circ) = E_\circ \sin 45^\circ \sin(kz - \omega t)$$

Which is a circularly polarized beam.

Problem 2.-Consider the wave described by:

$$\vec{E} = 4\hat{x}\cos(\omega t - kz) + 3\hat{y}\sin(\omega t - kz)$$

Indicate the state of polarization after going through a ¼ wave plate with the fast axis in the x direction.

Solution: If the beam described by the equation

$$\vec{E} = 4\hat{x}\cos(\omega t - kz) + 3\hat{y}\sin(\omega t - kz)$$

goes through a ¼ wave plate, both waves will be delayed, but the slow axis will be delayed more producing an effective lag of 90º, giving:

$$\vec{E} = 4\hat{x}\cos(\omega t - kz) + 3\hat{y}\sin(\omega t - kz - 90) = 4\hat{x}\cos(\omega t - kz) - 3\hat{y}\cos(\omega t - kz)$$

This is a plane polarized wave.

Problem 3.-Consider a half wave plate with the fast axis in the vertical direction illuminated with plane polarized light.
Determine the polarization state after going through the plate if the plane of polarization of the incident light makes and angle of 40º with the vertical axis.

Solution: Since the fast axis is vertical, then the horizontal component of the electric field will reverse after going through the plate. This is equivalent to an 80-degree rotation.