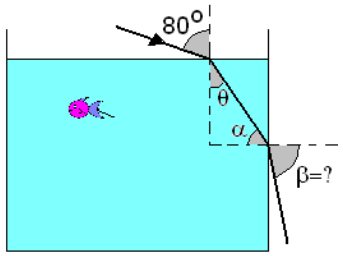
Optics

Snell's law

Snell's law $n_1 \sin\theta_1 = n_2 \sin\theta_2$

Total internal reflection $n_1 \sin\theta_{critical} = n_2 \sin 90^\circ$

Problem 1.- You point a laser beam to the surface of a fish tank as shown in the figure. Find the angle β at which the beam exits through the side of the tank (you can ignore the effect of the glass wall). The index of refraction of water is 1.33



Solution: To solve the problem we use Snell's law twice: at the first air-water interface:

$n_{air} Sin80^\circ = n_{air} Sin\theta$

$$\rightarrow (1)Sin80^\circ = 1.33 Sin\theta \rightarrow Sin\theta = \frac{Sin80^\circ}{1.33} = 0.7405 \rightarrow \theta = Sin^{-1}(0.7405) = \mathbf{47.8^o}$$

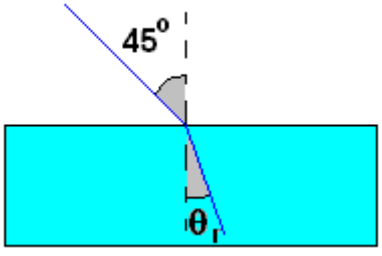
Now, according to the geometry of the problem the incident angle on the second water-air interface is $\alpha = 90 - \theta = 42.2^o$

And using Snell's law for a second time: $n_{water} Sin\alpha = n_{water} Sin\beta$

$$\rightarrow (1.33)Sin42.2^\circ = (1)Sin\beta \rightarrow Sin\beta = 0.8939 \rightarrow \beta = Sin^{-1}(0.8939) = \mathbf{63^o}$$

Problem 2.- You point a He-Ne laser (wavelength 633 nm in air) to the surface of a window with index of refraction n=1.58 at an angle of incidence of 45º. Calculate

a) The angle of refraction (θ_r).
b) The wavelength inside the window.



Solution:

a) The angle of refraction (θ_r):

$$\sin 45^\circ = 1.58 \sin\theta_r \rightarrow \theta_r = \sin^{-1}\left(\frac{\sin 45^\circ}{1.58}\right) = \mathbf{26.6^o}$$

b) The wavelength inside the window:

$$\lambda_{window} = \frac{\lambda_{air}}{1.58} = \frac{633nm}{1.58} = \mathbf{400.6\ nm}$$

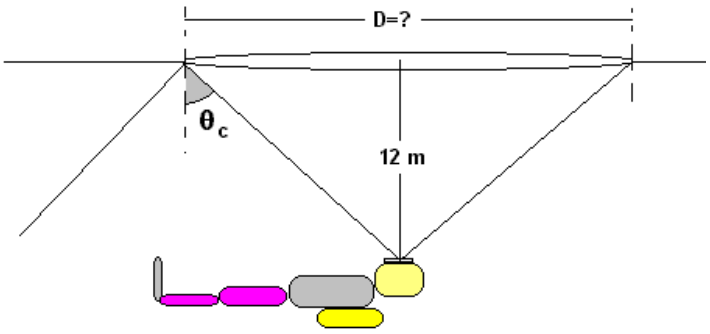
Problem 3.- A light source is at the bottom of a pool of alcohol (index of refraction is 1.36). At what minimum angle of incidence will a ray be totally reflected at the surface?

(A) 0º
(B) 22º
(C) 27º
(D) 47º
(E) 60º

Solution: To have total internal reflection the angle of refraction in air must be 90 degrees, so:

$$\sin 90^\circ = 1.36 \sin\theta \rightarrow \theta = \mathbf{47.3^o}$$

Problem 4.- A scuba diver observes a circle of light when looking to the surface of the water. This phenomenon is due to total internal reflection beyond the critical angle θ_c.
Calculate the diameter of the circle if the diver's depth is 12m.



Solution: The critical angle is:

$$\theta_c = \sin^{-1}\left(\frac{1}{1.33}\right) = 48.75^o$$

And the diameter then is:

$$D = 2 \times 12 \tan(48.75^\circ) = \mathbf{27.4\ m}$$