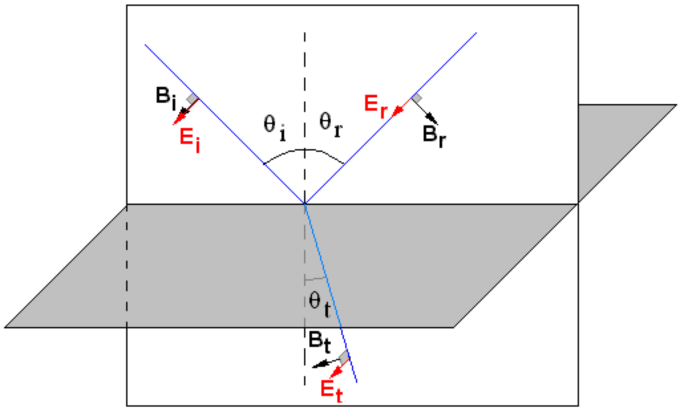
Optics

Fresnel equations

Case 1: TE mode: Consider a light ray that has polarization perpendicular to the plane of incidence as shown in the figure:



In this case, at the interface the boundary conditions are:

$$E_i + E_r = E_t$$

$$-\frac{B_i}{\mu_i}\cos\theta_i + \frac{B_r}{\mu_r}\cos\theta_i = -\frac{B_t}{\mu_t}\cos\theta_t$$

We can also consider the equations $B = \frac{E}{v}$ and $n = \frac{c}{v}$, so writing the magnetic field equation in terms of electric fields we get:

$$-\frac{n_i E_i}{\mu_i}\cos\theta_i + \frac{n_i E_r}{\mu_i}\cos\theta_i = -\frac{n_t E_t}{\mu_t}\cos\theta_t$$

$$\begin{pmatrix} 1 & -1 \\ \frac{n_i}{\mu_i}\cos\theta_i & \frac{n_t}{\mu_t}\cos\theta_t \end{pmatrix}\begin{pmatrix} E_r \\ E_t \end{pmatrix} = \begin{pmatrix} -1 \\ \frac{n_i}{\mu_i}\cos\theta_i \end{pmatrix} E_i$$

$$E_r = \frac{\begin{vmatrix} -1 & -1 \\ \frac{n_i}{\mu_i}\cos\theta_i & \frac{n_t}{\mu_t}\cos\theta_t \end{vmatrix}}{\begin{vmatrix} 1 & -1 \\ \frac{n_i}{\mu_i}\cos\theta_i & \frac{n_t}{\mu_t}\cos\theta_t \end{vmatrix}} E_i = \frac{-\frac{n_t}{\mu_t}\cos\theta_t + \frac{n_i}{\mu_i}\cos\theta_i}{\frac{n_t}{\mu_t}\cos\theta_t + \frac{n_i}{\mu_i}\cos\theta_i} E_i$$

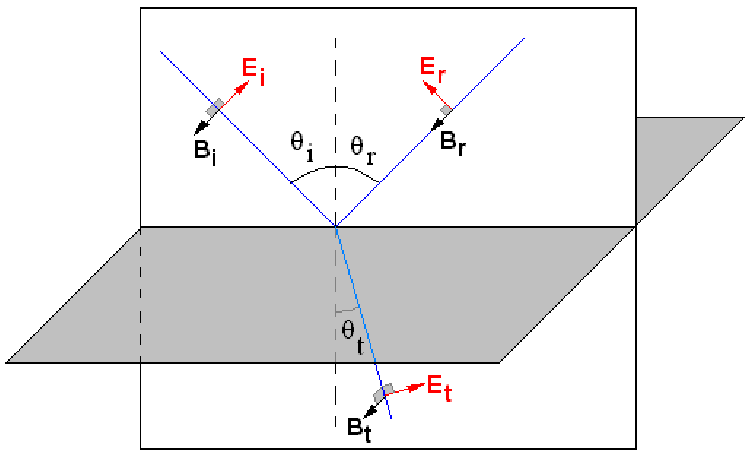
$$E_t = \frac{\begin{vmatrix} 1 & -1 \\ \frac{n_i}{\mu_i}\cos\theta_i & \frac{n_i}{\mu_i}\cos\theta_i \end{vmatrix}}{\begin{vmatrix} 1 & -1 \\ \frac{n_i}{\mu_i}\cos\theta_i & \frac{n_t}{\mu_t}\cos\theta_t \end{vmatrix}} E_i = \frac{2\frac{n_i}{\mu_i}\cos\theta_i}{\frac{n_t}{\mu_t}\cos\theta_t + \frac{n_i}{\mu_i}\cos\theta_i} E_i$$

In the common case where $\mu_i = \mu_t = \mu_o$, the equations reduce to:

$$\frac{E_r}{E_i} = \frac{n_i \cos\theta_i - n_t \cos\theta_t}{n_i \cos\theta_i + n_t \cos\theta_t}$$

$$\frac{E_t}{E_i} = \frac{2n_i \cos\theta_i}{n_i \cos\theta_i + n_t \cos\theta_t}$$

Case 2 TM Mode: Consider a light ray that has polarization parallel to the plane of incidence as shown in the figure:



For this light ray, at the interface the boundary conditions are:

$$E_i \cos\theta_i - E_r \cos\theta_r = E_t \cos\theta_t$$

$$\frac{B_i}{\mu_i} + \frac{B_r}{\mu_r} = \frac{B_t}{\mu_t}$$

Rewriting the magnetic equations in terms of electric fields and index of refraction:

$$\frac{n_i E_i}{\mu_i} + \frac{n_i E_r}{\mu_i} = \frac{n_t E_t}{\mu_t}$$

Writing the two equations in matrix form we get:

$$\begin{pmatrix} \dfrac{n_i}{\mu_i} & -\dfrac{n_t}{\mu_t} \\ \cos\theta_i & \cos\theta_t \end{pmatrix} \begin{pmatrix} E_r \\ E_t \end{pmatrix} = \begin{pmatrix} -\dfrac{n_i}{\mu_i} \\ \cos\theta_i \end{pmatrix} E_i$$

solving for the electric field:

$$E_r = \frac{\begin{vmatrix} -\frac{n_i}{\mu_i} & -\frac{n_t}{\mu_t} \\ \cos\theta_i & \cos\theta_t \end{vmatrix}}{\begin{vmatrix} \frac{n_i}{\mu_i} & -\frac{n_t}{\mu_t} \\ \cos\theta_i & \cos\theta_t \end{vmatrix}} E_i = \frac{\frac{n_t}{\mu_t}\cos\theta_i - \frac{n_i}{\mu_i}\cos\theta_t}{\frac{n_i}{\mu_i}\cos\theta_t + \frac{n_t}{\mu_t}\cos\theta_i} E_i$$

and:

$$E_t = \frac{\begin{vmatrix} \frac{n_i}{\mu_i} & -\frac{n_i}{\mu_i} \\ \cos\theta_i & \cos\theta_i \end{vmatrix}}{\begin{vmatrix} \frac{n_i}{\mu_i} & -\frac{n_t}{\mu_t} \\ \cos\theta_i & \cos\theta_t \end{vmatrix}} E_i = \frac{2\frac{n_i}{\mu_i}\cos\theta_i}{\frac{n_i}{\mu_i}\cos\theta_t + \frac{n_t}{\mu_t}\cos\theta_i} E_i$$

The most common case is that $\mu_i = \mu_t = \mu_o$, so the equations reduce to:

$$\frac{E_r}{E_i} = \frac{n_t\cos\theta_i - n_i\cos\theta_t}{n_t\cos\theta_i + n_i\cos\theta_t}$$

$$\frac{E_t}{E_i} = \frac{2n_i\cos\theta_i}{n_i\cos\theta_t + n_t\cos\theta_i}$$