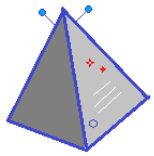
Physics I

Blackbody Radiation

Radiation law: $\text{Radiation}(power) = Area \times \varepsilon\sigma T^4$, where $\sigma = 5.67\times10^{-8}\ \frac{\text{W}}{\text{m}^2\text{K}^4}$

Problem 1.- Consider a satellite with the shape of a tetrahedron and with one face directed towards the sun receiving 1350 W/m^2 of radiation. Calculate the temperature of the satellite if all 4 faces are in equilibrium.



Solution: The satellite receives energy at a rate equal to:

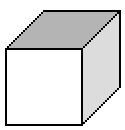
$P_{absorbed} = 1{,}350 \times A$, where A is the area of one of the triangles.

but emits in all directions:

$P_{emitted} = \sigma T^4 \times (4A)$

These quantities should be the same in equilibrium, so: $T = \sqrt[4]{\frac{1350}{5.67\times10^{-8}\times 4}} =$ **278 K**

Problem 2.- Consider that a satellite in orbit has the shape of a cube and one side is facing the sun and receiving 1,300 W/m^2 of radiation. Calculate the temperature of the satellite if it emits equally from all its 6 sides.



Solution: The satellite receives energy at a rate equal to: $P_{absorbed} = 1{,}300 \times L^2$,

but emits in all directions: $P_{emitted} = \sigma T^4 \times (6L^2)$

These quantities should be the same in equilibrium, so:

$1{,}300 \times L^2 = \sigma T^4 \times (6L^2) \rightarrow T = \sqrt[4]{\frac{1{,}300}{5.67\times10^{-8}\times 6}} =$ **250 K**

Problem 3.- Planet Mercury has a spherical shape. It receives radiation from the sun at a rate of 9,300 W/m^2. Estimate the temperature of the surface assuming it behaves like a black body with constant temperature.

Hint: *Consider that the sun only illuminates an area equivalent to a circle, but the planet emits in every direction.*

Solution: The energy absorbed is: $area \times 9300 = \pi r^2 \times 9300$

The energy emitted is: $area \times \sigma T^4 = 4\pi r^2 \times \sigma T^4$

These two are equal in equilibrium, so: $\pi r^2 \times 9300 = 4\pi r^2 \times \sigma T^4 \rightarrow T = \sqrt[4]{\frac{9300}{4\sigma}} =$ **450 K**

Problem 4.- Why is a good emitter of radiation called a black body?

Answer: Good emitters are also good absorbers, so they look black at room temperature.

Problem 5.- Consider that a satellite in orbit has the shape of a cylinder with the circular base facing the sun and receiving 1,350 W/m^2 of radiation. Calculate the temperature of the satellite if it emits equally from all parts of its surface and has a length of 20R (R being the radius of the base).

Solution: The satellite receives energy at a rate equal to

$$P_{absorbed} = 1350 \times \pi R^2,$$

but emits in all directions:

$$P_{emitted} = \sigma T^4 \times \left(2\pi R^2 + 2\pi R \times 20R\right)$$

These quantities should be the same in equilibrium, so

$$1350 \times \pi R^2 = \sigma T^4 \times \left(2\pi R^2 + 2\pi R \times 20R\right) \rightarrow T = \sqrt[4]{\frac{1350 \times \pi R^2}{5.67 \times 10^{-8} \times 42\pi R^2}} = \mathbf{154\ K}$$

Problem 6.- The Sun, whose surface temperature is 5800K, emits 3.2×10^{26} W of radiation. Consider a brown dwarf star with the same size, but which only emits 0.2×10^{26} W. Calculate its surface temperature.

Solution: Since the brown dwarf radiation is 1/16 of the Sun and the radiation is proportional to the temperature to the fourth power, the temperature of the brown dwarf will be ½ the temperature of the sun (**2,900 K**).

Problem 7.- Regarding the density of energy in a cavity as a function of lambda $U(\lambda)$ and as a function of frequency $U(f)$:

A) Why is the maximum given by Wien's law, $\lambda_{MAX} T = 2.9\text{mmK}$, not the same for U(f)?

B) Use the law stated above to find the wavelength at maximum intensity produced by a pizza oven whose temperature is 220ºC

Solution:

A) Since the differentials dλ and df are not equal, the maximum given by Wien's law is not the same for U(f). This is because when taking the differential df you pick up a factor $-c/\lambda^2$.

B) T = 220+273.15 = 493.15K, so: $\lambda_{MAX} = \frac{2.9\text{mmK}}{493.15\text{K}} = \mathbf{5.88\ \mu m}$