

# Physics I

## Boltzmann factors

The probability of being in a state with energy  $\epsilon$  is proportional to the Boltzmann factor  $e^{-\frac{\epsilon}{k_B T}}$  where  $k_B = 1.38 \times 10^{-23} \text{ J/K}$

**Problem 1.-** A paramagnetic atom has two states with energies  $E_1 = 0 \text{ J}$ , which is the ground state, and  $E_2 = 2.87 \times 10^{-21} \text{ J}$ , which is the excited state.

Calculate the probability of being in the ground state when  $T = 300 \text{ K}$ .

**Problem 2.-** Suppose that an atom has a ground state with energy zero and three excited states with energy  $\epsilon$ . The atom is in an environment where the thermal energy  $k_B T$  is much larger than  $\epsilon$ . Estimate the probability of finding the atom in the ground state.

**Problem 3.-** Which of the following molecules moves fastest in the atmosphere and which is the slowest and why?

Atomic masses: H=1; C=12; N=14; O=16

H<sub>2</sub>O

CO<sub>2</sub>

O<sub>2</sub>

N<sub>2</sub>

**Problem 4.-** What is the probability of finding the spin of a free electron in its ground state when the magnetic field is  $B = 1.0 \text{ tesla}$ , and the temperature is  $T = 1.5 \text{ K}$ ?

The product of the magnetic moment times the magnetic field is:  $\mu B = 9.3 \times 10^{-24} \text{ J}$

**Problem 5.-** The Maxwell distribution of gas speeds is given by:

$$f(v) = 4\pi N \left( \frac{m}{2\pi k_B T} \right)^{3/2} v^2 e^{-\frac{1}{2} \frac{mv^2}{k_B T}}$$

Calculate the most probable speed, which is the speed that has the maximum probability of occurring.

**Problem 6.-** Calculate the ratio of rms speed between SF<sub>6</sub> molecules and He atoms at room temperature if they are diluted enough to treat them as ideal gases.

Atomic mass of S = 32, and F = 19

**Problem 7.-** Calculate the probability to find a diatomic molecule in its vibrational ground state if the temperature is 300K and the first excited state has an energy of  $6.9 \times 10^{-21} \text{ J}$  above the ground state.

To simplify the problem, consider only two states: the ground state with energy  $E_1=0$  and the first excited state.

**Problem 8.-** Helium doesn't stay in the atmosphere very long because it has such a light mass that it can easily leave the surface of the Earth. Calculate the average speed (the rms value) of helium at  $T=300\text{K}$ .

**Problem 8a.-** Calculate the rms average speed of the amino acid Glycine in the gas phase at  $37^\circ\text{C}$  knowing that its mass is 75 amu and it can be treated as an ideal gas.  
 $1\text{amu}=1.66\times 10^{-27}\text{ kg}$

**Problem 8b.-** In principle, can we separate  $\text{N}_2$  from  $\text{O}_2$  by diffusion? What is the ratio of speeds of these two molecules at room temperature?

**Problem 9.-** Calculate the probability that an electron will be in the conduction band of silicon, whose energy is 1.12 eV higher than the valence band. Take  $T=300\text{K}$  and to simplify the problem consider only two states: the ground state with energy  $E_1=0$  and the first excited state with  $E_2=1.12\text{eV}$ .

$$1\text{ eV}=1.6\times 10^{-19}\text{ J}$$