

Physics I

Latent Heat

Problem 1.- A 10-gram bullet made from an alloy is found to have melted on impact with an armored truck. Assuming the bullet was fired at room temperature (20° C) estimate the minimum muzzle velocity of the bullet.

Recall that kinetic energy is $KE = \frac{1}{2}mv^2$

Specific heat capacity of the alloy $c = 120 \frac{J}{K \cdot kg}$

Melting point of the alloy $T = 330^\circ C$

Latent heat of the alloy $L = \frac{24,000J}{kg}$

Solution: To totally melt the bullet, we need to increase its temperature to the melting point and then change the phase to liquid, as follows:

$$Q = mc\Delta T + mL = 0.01 \times 120 \times (330 - 20) + 0.01 \times 24,000 = 612J$$

That energy comes from the kinetic energy, so: $612 = \frac{1}{2}mv^2 \rightarrow v = \sqrt{\frac{2 \times 612}{0.01}} = \mathbf{350 \text{ m/s}}$

Problem 1a.- How fast would a silver bullet had to go to melt on impact assuming all the kinetic energy goes to the bullet?

Consider the initial temperature $T = 20^\circ C$

Melting point of silver = 961 °C

Latent heat of fusion of silver = 88J/g, specific heat of silver = 0.23J/gram °C

Solution: Like the previous problem, suppose you have 1kg of silver. You need to heat the silver to the melting point first:

$$Q_1 = cm\Delta T = 0.23 \frac{J}{g^\circ C} \times 1000g \times (961 - 20) = 216,000J$$

And then you need to melt it:

$$Q_2 = Lm = 88 \frac{J}{g} \times 1000g = 88,000J$$

So the total will be **304,000 J**

This energy was originally kinetic energy, so:

$$Q = \frac{1}{2}mv^2 \rightarrow v = \sqrt{\frac{2Q}{m}} = \sqrt{2 \times 304,000} = \mathbf{779 \text{ m/s}}$$

Problem 2.- Is it possible to boil water at room temperature (27°C)? Explain.

Solution: It is possible to boil water at room temperature (27°C), by reducing the external pressure to the vapor pressure that corresponds to 27°C.

Problem 3.- You add a cube of ice (mass 8 grams) at $T=0^{\circ}\text{C}$ to a 300-gram drink at $T=18^{\circ}\text{C}$ (consider the drink to be mainly water).

Calculate the final temperature of the mixture assuming no heat is transferred to the surroundings.

Latent heat of fusion of ice 80cal/g

Specific heat of water $1\text{cal/gram }^{\circ}\text{C}$

Solution: The heat necessary to change the phase of the ice is:

$$Q_1 = Lm = 80 \frac{\text{cal}}{\text{g}} \times 8\text{g} = 640\text{cal}$$

Then the ice will be heated to reach its final temperature T , so:

$$Q_2 = cm\Delta T = 1 \frac{\text{cal}}{\text{g}^{\circ}\text{C}} \times 8\text{g} \times (T - 0) = 8T\text{cal}$$

On the other hand the drink will cool down from 18°C to a final temperature T , releasing this amount of heat:

$$Q_3 = cm\Delta T = 1 \frac{\text{cal}}{\text{g}^{\circ}\text{C}} \times 300\text{g} \times (18 - T) = 300(18 - T)\text{cal}$$

Now, the amount of heat lost by the drink will be gained by the ice, so:

$$300(18 - T) = 640 + 8T \rightarrow 5400 - 300T = 640 + 8T \rightarrow T = \frac{5400 - 640}{300 + 8} = \mathbf{15.5^{\circ}\text{C}}$$

Problem 3a.- How much ice at $T=0^{\circ}\text{C}$ do you need to add to a 300-gram drink at $T=18^{\circ}\text{C}$ (consider the drink to be mainly water) to cool it down to $T=0^{\circ}\text{C}$?

Assume no heat is transferred to the surroundings.

Latent heat of fusion of ice 80cal/g

Specific heat of water $1\text{cal/g }^{\circ}\text{C}$

Solution: The ice will change phase absorbing $80m$ calories and that has to match the heat lost by the drink, so:

$$80m = 300 \times \frac{1\text{cal}}{\text{g}^{\circ}\text{C}} \times 18^{\circ}\text{C} \rightarrow m = \frac{300 \times 18}{80} = \mathbf{67.5\text{g}}$$