Physics I

Thermodynamic Cycles

Carnot cycle efficiency $\eta = 1 - \dfrac{T_{Low}}{T_{high}}$

Problem 1.- An ideal gas initially occupies a volume of 2L at a pressure of 150,000 pascals. It expands isothermally to a volume of 5L.
How much work is done in the process?
Recall that $\int_{x1}^{x2} \frac{dx}{x} = \ln\left(\frac{x_2}{x_1}\right)$

Solution: $W = \int_{V1}^{V2} pdV = \int_{V1}^{V2} \frac{P_1V_1}{V} dV = P_1V_1 \ln\left(\frac{V_2}{V_1}\right) = 150{,}000 \times 0.002 \ln(2.5) =$ **275 J**

Problem 1a.- Calculate the work delivered by the isothermal expansion of 5.8 kg of air at T=600 K from an initial pressure of p_1 = 8 atm to a final pressure of p_2 = 2 atm.
Approximate air as an ideal gas of molecular mass 29.

Solution: $Work = \int_{V1}^{V2} pdV = \int_{V1}^{V2} \frac{nRT}{V} dV = nRT \int_{V1}^{V2} \frac{dV}{V} = nRT \ln\left(\frac{V_2}{V_1}\right)$

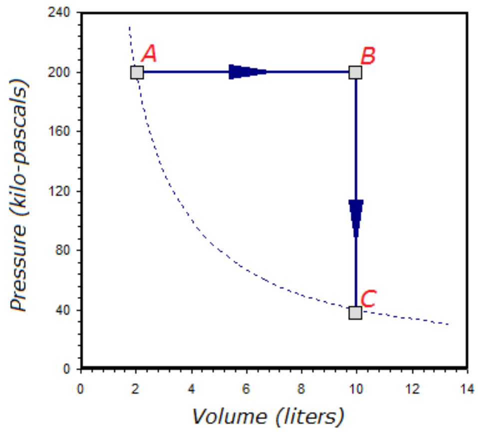
The problem indicates that the final pressure is ¼ the initial pressure, but this is an ideal gas at constant temperature, so the final volume is 4 times the initial volume.

The number of moles is: $n = \frac{mass(in\ grams)}{Molecular\ mass} = \frac{5800}{29} = 200$ moles

The work done is then: $Work = nRT \ln\left(\frac{V_2}{V_1}\right) = 200 \times 8.314 \times 600 \ln(4) =$ **1.38 MJ**

Problem 2.- An ideal gas initially occupies a volume of 2L at a pressure of 200,000 pascals. It expands at constant pressure to a volume of 10L. Then it is cooled down at constant volume until its final temperature is equal to the initial temperature.
Sketch the process in a PV diagram with units and values.
How much work is done in the process?

Solution: The figure sketches the process. Notice that the initial point A and the final point C have the same temperature and so they belong to the isothermal shown as a dotted line. The isothermal curve is a hyperbola, a conic section.



The work is done in the A to B part:

$W = pressure \times \Delta V = 200{,}000 \times 0.008 =$ **1,600 J**

Problem 3.- A mixture of gasses is found experimentally to have a heat capacity at constant pressure of $C_v = 2R$ per mole, so $C_p = 3R$

Knowing that $P_1 V_1^{\gamma} = P_2 V_2^{\gamma}$ calculate the final pressure of the mixture if it expands adiabatically from an initial pressure of 1 atm and volume 1L to a final volume of 2L.

Solution: Knowing that $P_1 V_1^{\gamma} = P_2 V_2^{\gamma}$

$1 \times 1^{1.5} = P_2 \times 2^{1.5} \rightarrow P_2 = \frac{1}{2^{1.5}} =$ **0.35 atm**

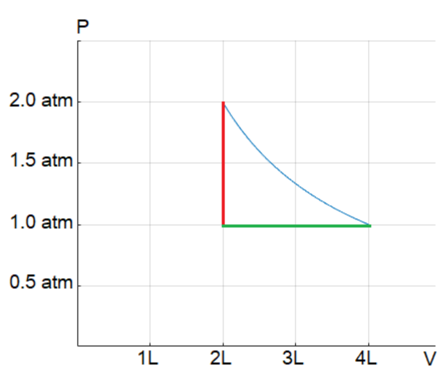
Problem 4.- Draw a P-V diagram of the following cycle:

a) A 2L volume of air initially at 2 atm expands isothermally to a final volume of 4L.

b) The gas is then compressed at constant pressure to a final volume of 2L

c) The gas is heated at constant volume until it reaches a pressure of 2 atm again.

Solution:



Problem 5.- An ideal gas initially occupies a volume of 2L at a pressure of 200,000 pascals. It expands isothermally to a volume of 10L.
How much work is done in the process?

$$\int_{x1}^{x2} \frac{dx}{x} = \ln\left(\frac{x_2}{x_1}\right)$$

Solution: $W = \int_{V1}^{V2} pdV = \int_{V1}^{V2} \frac{P_1V_1}{V} dV = P_1V_1 \ln\left(\frac{V_2}{V_1}\right) = 200{,}000 \times 0.002 \ln(5) =$ **644 J**

Problem 6.- Calculate the amount of work done by air expanding from an initial temperature of 800K and volume of 0.15 m^3 to a final volume of 0.25 m3 in an adiabatic expansion (gamma for air is 1.4)

Solution: let's first find the initial pressure

$$P_1V_1 = nRT_1 \rightarrow P_1 = \frac{nRT_1}{V_1}$$

And the final pressure

$$P_1V_1^{\gamma} = P_2V_2^{\gamma} \rightarrow P_2 = P_1\left(\frac{V_1}{V_2}\right)^{\gamma}$$

The work done is $\int_{V1}^{V2} pdV = \int_{V1}^{V2} \frac{p_1V_1^{\gamma}}{V^{\gamma}} dV = p_1V_1^{\gamma} \int_{V1}^{V2} \frac{dV}{V^{\gamma}} = \left. \frac{p_1V_1^{\gamma}V^{-\gamma+1}}{-\gamma+1} \right|_{V1}^{V2}$

$$= \frac{p_1V_1^{\gamma}V_2^{-\gamma+1}}{-\gamma+1} - \frac{p_1V_1^{\gamma}V_1^{-\gamma+1}}{-\gamma+1} = \frac{p_1V_1 - p_2V_2}{\gamma-1} = \frac{\frac{nRT_1}{V_1}\left[1-\left(\frac{V_1}{V_2}\right)^{\gamma-1}\right]}{\gamma-1}$$

$$= \frac{\frac{nRT_1}{V_1}\left[1-\left(\frac{V_1}{V_2}\right)^{\gamma-1}\right]}{\gamma-1} = \frac{\frac{n8.314\times800}{0.15}\left[1-\left(\frac{0.15}{0.25}\right)^{0.4}\right]}{0.4} = \mathbf{20{,}500\ n\ joule}$$

Problem 7.- If an engine works between a maximum temperature of 900K and a minimum of 300K, what is the theoretical upper bound of the efficiency?

Solution: $\eta = 1 - \frac{300}{900} = \mathbf{67\%}$