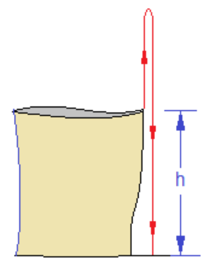
Physics I

Falling Objects

In case of free fall, the acceleration in the vertical direction is a_y=-9.8m/s^2

$$y = v_{y1}t + \frac{1}{2}a_y t^2 \qquad v_{y2} = v_{y1} + a_y t \qquad v_{y2}{}^2 = v_{y1}{}^2 + 2a_y y \qquad \langle v_y \rangle = \frac{v_{y1} + v_{y2}}{2} = \frac{y}{t}$$

Problem 1.- A stone is thrown straight upwards with an initial speed of 15 m/s at the edge of a cliff whose height is h=65m. Calculate the time it will take for the stone to reach the bottom of the cliff.



Solution: There are several ways to solve this problem, for example we could use the equation $y = v_{y1}t + \frac{1}{2}a_y t^2$ since we already know $y = -65m$, $v_{y1} = 15m/s$ and $a_y = -9.8m/s^2$ then we get $-65 = 15t - 4.9t^2$, which is a quadratic equation, that we can solve using the well know quadratic formula $4.9t^2 - 15t - 65 = 0 \rightarrow t = \frac{-(-15) \pm \sqrt{15^2 - 4(4.9)(-65)}}{2(4.9)} = \mathbf{5.5\ s}$

An alternative approach would be to find v_{y2} first and then using that in the equation $v_{y2} = v_{y1} + a_y t$ to calculate the time.

Problem 2.- A stone is thrown vertically upwards with an initial velocity of 25 m/s. Determine:

i) The maximum height reached and
ii) How long it takes to get there.

Solution: At the maximum height reached the velocity is zero for an instant, so knowing that $v_2 = 0$ we have:

i) $v_2{}^2 = v_1{}^2 + 2ax \rightarrow 0^2 = 25^2 + 2(-9.8)x \rightarrow x = \frac{25^2}{2 \times 9.8} = \mathbf{32\ m}$

ii) $v_2 = v_1 + at \rightarrow 0 = 25 + (-9.8)t \rightarrow t = \frac{25}{9.8} = \mathbf{2.6\ s}$

Problem 3.- Neglecting air resistance, estimate the time it would take a penny to fall straight down from the top of the Empire State Building (380 m high), and its velocity just before hitting the ground. Assume zero initial velocity.

Solution: First, let us find the time:

$$380 = v_1 t + \frac{1}{2}at^2 = \frac{1}{2}9.8t^2 \rightarrow t = \sqrt{\frac{380}{4.9}} = \textbf{8.8 s}$$

To find the final velocity:

$$v_2 = v_1 + at = 9.8 \times 8.8 = \textbf{86 m/s}$$

Problem 3a.- Neglecting air resistance, estimate how long it would take a penny to fall straight down from the top of the Eiffel Tower (324 m high), and its velocity just before hitting the ground. Assume the initial velocity to be zero.

Solution: Using the equation $x = v_1 t + \frac{1}{2}at^2$ we get:

$$-324 = 0t + \frac{1}{2}(-9.8)t^2 \rightarrow t = \sqrt{\frac{324 \times 2}{9.8}} = \textbf{8.1 s}$$

And the velocity will be $v_2 = v_1 + at = 0 \times 8.1 + (-9.8) \times 8.1 =$ **-80 m/s**

Problem 4.- A stone is thrown vertically upwards with an initial velocity of 29.6 m/s. Determine its velocity when it reaches a height of 18 m.

Solution: We use $v_2{}^2 = v_1{}^2 + 2ax$ to solve the problem:

$$v_2{}^2 = 29.6^2 + 2(-9.8)(18) \rightarrow v_2 = \textbf{23 m/s}$$

Problem 5.- Two projectiles are shot straight up with initial velocities of 30 m/s, but with 1s of delay between them. At what height will the projectiles hit each other?

Solution: We use the equation: $x = v_1 t + \frac{1}{2}at^2$

The height of the first projectile will be: $x = 30t - 4.9t^2$

The height of the second projectile will be: $x = 30(t-1) - 4.9(t-1)^2$

Since the heights should be the same, we get:

$$30t - 4.9t^2 = 30(t-1) - 4.9(t-1)^2$$

Solving for t:

$$30t - 4.9t^2 = 30t - 30 - 4.9(t^2 - 2t + 1)$$
$$0 = -30 - 4.9(-2t + 1)$$
$$t = 3.56s$$

Using this time we find the height: $x = 30t - 4.9t^2 = 30 \times 3.56 - 4.9 \times 3.56^2 =$ **44.7 m**

Alternative Solution: Notice that when the two projectiles collide their velocities will be equal in magnitude, but in opposite directions. You can tell that is the case if you recall that $v_2^2 = v_1^2 + 2ax$, so at the point of collision:

$v_{2-\text{first\_particle}} = -v_{2-\text{second\_particle}}$

Now using the equation $v_2 = v_1 + at$ we can write:

$$30 - 9.8t = -[30 - 9.8(t-1)]$$

And solving for t we get: $30 - 9.8t = -30 + 9.8t - 9.8 \rightarrow 19.6t = 69.8 \rightarrow t = 3.56$

And the height is then: $x = v_1 t + \frac{1}{2}at^2 = 30 \times 3.56 - \frac{1}{2}9.8 \times 3.56^2 =$ **44.7 m**

Problem 6.- To measure the height of a building you drop a marble from the roof and measure the time it takes to hit the ground. Calculate the height of the building if the time is t=4.95 s

Solution: Since we neglect air resistance, this is a case of free fall. The equation that describes the position is: $y = v_{1y}t + \frac{1}{2}a_y t^2$

With t=4.95 s we get: $y = v_{1y}t + \frac{1}{2}a_y t^2 = (0)(4.95) + \frac{1}{2}(-9.8)(4.95)^2 =$ **-120 m**

Problem 7.- In an experiment you determine that at a certain point the velocity of an object is 1.5 m/s. Knowing that its initial velocity was zero calculate:

a) How long it has been falling (time, t).
b) How much it has fallen (distance, y).

Solution:

a) $v_2 = v_1 + (-9.8)t \rightarrow t = \frac{1.5}{9.8} =$ **0.15 s**

b) $v_2^2 = v_1^2 + 2(-9.8)y \rightarrow y = \frac{v_2^2 - v_1^2}{2(9.8)} = \frac{1.5^2}{2(9.8)} =$ **0.11m**