Physics I

Projectiles

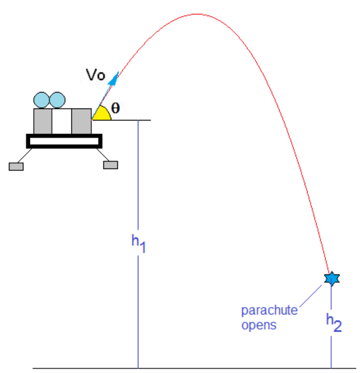
In free fall $x = v_x t$ in the horizontal direction and in the vertical direction:

$$y = v_{y1}t + \frac{1}{2}a_y t^2 \qquad v_{y2} = v_{y1} + a_y t \qquad {v_{y2}}^2 = {v_{y1}}^2 + 2a_y y \qquad \langle v_y \rangle = \frac{v_{y1} + v_{y2}}{2} = \frac{y}{t}$$

Range of a projectile thrown at ground level: $R = \dfrac{V_o^2 \sin(2\theta)}{g}$

Problem 1.- In an emergency an astronaut ejects from a lunar landing simulator at a height h_1=75m and an initial velocity $V_o = 24$m/s that makes an angle $\theta = 53^\circ$ with the horizontal.

Calculate how much time there is to open the parachute knowing that it has to be opened at least h_2=25m above the ground.

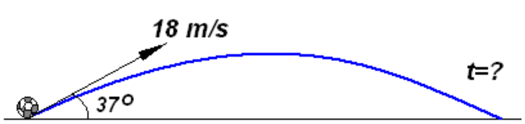


Solution: The initial velocity in the vertical direction is $V_{y1} = 24\sin 53^\circ = 19.1 m/s$ and the vertical displacement is $y = -50m$, which we can use to find the time:

$$-50 = 19.1t + \frac{1}{2}(-9.8)t^2 \rightarrow 4.9t^2 - 19.1t - 50 = 0$$

$$\rightarrow t = \frac{19.1 + \sqrt{19.1^2 - 4(4.9)(-50)}}{2 \times 4.9} = \mathbf{5.7\ s}$$

Problem 2.- A football is kicked at ground level with a speed of 18.0 m/s and at an angle of 37.0 degrees to the horizontal. Calculate the range, how long it is in the air, the maximum height reached and the velocity at the top of the trajectory.



Solution:

We find the range first: $R = \frac{V_o^2 \sin(2\theta)}{g} = \frac{18^2 \sin(2\times 37^o)}{9.8} = 31.8$ m

And to find the time we use the horizontal equation:

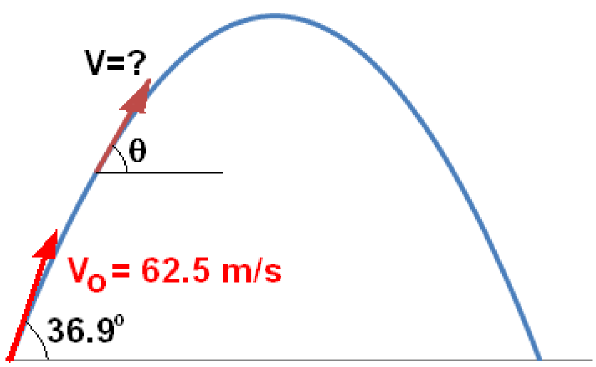
$$\mathrm{x} = \mathrm{v_x t} \rightarrow \mathrm{t} = \frac{\mathrm{x}}{\mathrm{v_x}} = \frac{31.8}{18\cos 37^{\circ}} = \mathbf{2.2\ s}$$

To find the maximum height notice that at that point the vertical velocity is zero, so we can use the equation $v_{y2}{}^2 = v_{y1}{}^2 + 2a_y y$ to find y:

$0^2 = 10.8^2 + 2(-9.8)y \rightarrow y = \mathbf{6.0\ m}$

The velocity at the top of the trajectory is (18cos37º,0) since the vertical velocity is zero at that point.

Problem 3.- A projectile is fired with an initial speed of 62.5 m/s at an angle of 36.9º above the horizontal on a long flat firing range. Determine the velocity (in magnitude and angle) 1.25 seconds after firing [neglect air resistance].



Solution: The initial velocity is given by the two components:

$V_{x1} = 62.5\cos 36.9^{\circ} = 50$ m/s and $V_{y1} = 62.5\sin 36.9^{\circ} = 37.5$ m/s

The velocity in the x-direction will stay the same, but the velocity in the y-direction will change according to the equation:

$V_{y2} = V_{y1} + a_y t = 37.5 + (-9.8)t$

So, if the time is **t=1.25** seconds, the velocity in the y-direction will be:

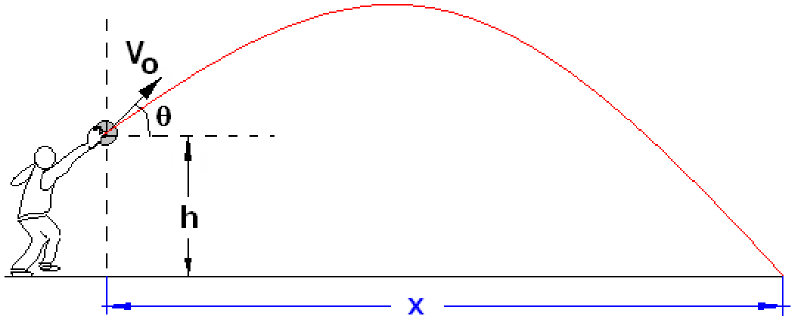
$V_{y2} = 37.5 + (-9.8)(1.25) = 25.3$ m/s

The magnitude of the velocity at that time will be:

$V = \sqrt{50^2 + 25.3^2} = \mathbf{56\ m/s}$

And the angle of the vector will be: $\theta = \tan^{-1}\left(\dfrac{25.3}{50}\right) = \mathbf{26.8^o}$

Problem 4.- A shot-putter throws the shot with an initial speed of $\mathbf{V_o=14.5}$ m/s at an angle $\mathbf{\theta=35.5^o}$ to the horizontal. Calculate the horizontal distance traveled by the shot if it leaves the athlete's hand at a height of **h= 2.15** m above the ground.



Solution: The initial velocity of the shot is given by the two components:

$V_{x1} = 14.5\cos 35.5^\circ = 11.8$ m/s

$V_{y1} = 14.5\sin 35.5^\circ = 8.4$ m/s

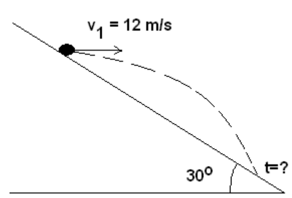
We can use the y-direction to find the time: $y = v_{1y}t + \dfrac{1}{2}a_y t^2$

$y = 8.4t - 4.9t^2 \rightarrow 4.9t^2 - 8.4t + y = 0$

So the time is: $t = \dfrac{8.4 + \sqrt{8.4^2 - 4(4.9)y}}{2\times 4.9}$

If y=-2.15 the time is t=1.94 and $x = v_x t = (11.8)(1.94) = \mathbf{22.9\ m}$

Problem 5.- A person kicks a rock horizontally as shown in the figure. Calculate the time it takes to hit the ground.



Solution: Notice that the hill corresponds to the equation $y = -x\tan 30^\circ$

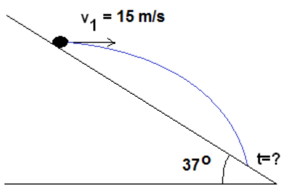
Or more simply: $y = -0.577x$

But $x = v_x t = 12t$ and $y = v_{y1}t + \frac{1}{2}a_y t^2 = -4.9t^2$, so replacing these values in the equation above:

$$-4.9t^2 = -0.577(12t)$$

This last equation has two solutions: t=0 and $t = \frac{0.577(12)}{4.9} =$ **1.4 s**

Problem 5a: A person kicks a rock horizontally as shown in the figure. Calculate the time it takes to hit the ground.



Solution: Notice that the hill corresponds to the equation $y = -x \tan 37^\circ$

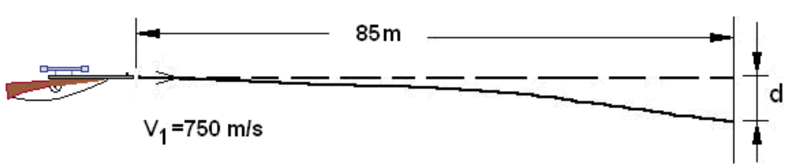
Or more simply: $y = -0.75x$

But $x = v_x t = 15t$ and $y = v_{y1}t + \frac{1}{2}a_y t^2 = -4.9t^2$, so replacing these values in the equation above:

$$-4.9t^2 = -0.75(15t)$$

This last equation has two solutions: the trivial $t = 0$ and $t = \frac{0.75(15)}{4.9} =$ **2.3 s**

Problem 6.- A shooter knows that if she aims directly at a target, which is at the same level, she'll miss (as shown in the figure). Calculate the angle that she should use to correct the trajectory and hit the target.
[Neglect air resistance]

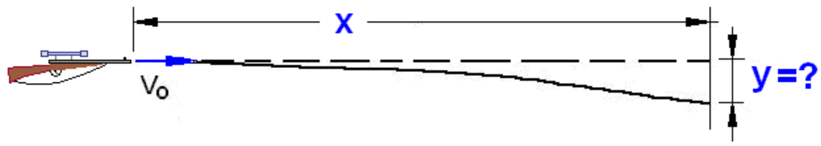


Solution: We can use the "range" equation to solve the problem: $R = \frac{V_o^2 Sin(2\theta)}{g}$

$$85 = \frac{750^2 Sin(2\theta)}{9.8} \rightarrow \sin 2\theta = \frac{85 \times 9.8}{750^2} = 0.00144$$

$2\theta = \sin^{-1}(0.00144) = 0.08^\circ \rightarrow \theta = \mathbf{0.04^o}$

Problem 6a.- A shooter aims directly at a target which is at the same level, a distance $x = 120m$ away. If the bullet leaves the gun at a speed $v_o = 650m/s$, by how much will it miss the target? [Neglect air resistance]

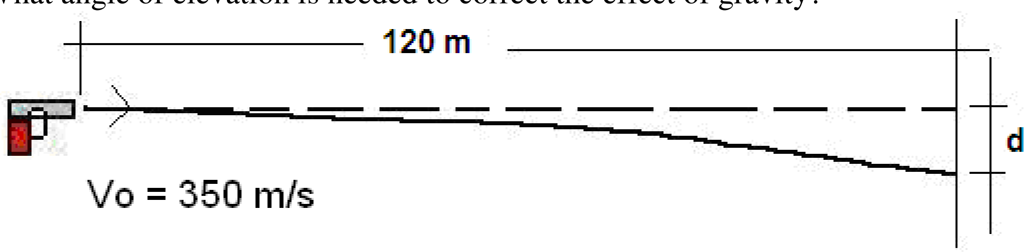


Solution: We can focus on the x-axis to find the time it takes the bullet to hit the target:
$x = v_{x1}t \rightarrow 120 = 650t \rightarrow t = 0.185s$

Now we can use this time to find how much the bullet dropped: (equation in the y-axis)
$$y = v_{y1}t + \frac{1}{2}a_y t^2 = \frac{1}{2}(\text{-}9.8\text{m/s}^2)(0.185\text{s})^2 = \mathbf{-0.17\ m}$$

Problem 6b.- A shooter aims directly at a target which is at the same level, 120 m away.

a) Calculate by how much he will miss
b) What angle of elevation is needed to correct the effect of gravity?

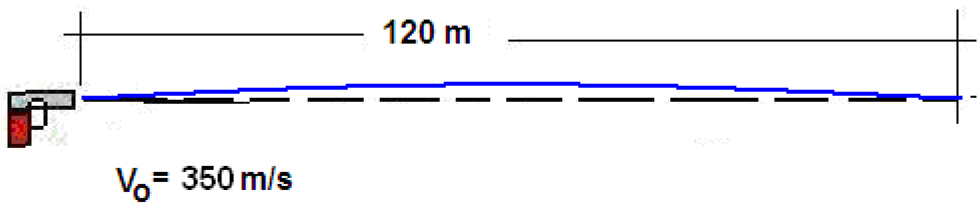


Solution:

a) The time it takes the bullet to hit the target is $t = \dfrac{120m}{350m/s} = 0.343s$ and it drops:

$$y = \frac{1}{2}a_y t^2 = \frac{1}{2}(-9.8m/s^2)(0.343s)^2 = \mathbf{0.58m}$$

b) If you recognize that this problem is like a "range problem" then you can use the equation for "R":

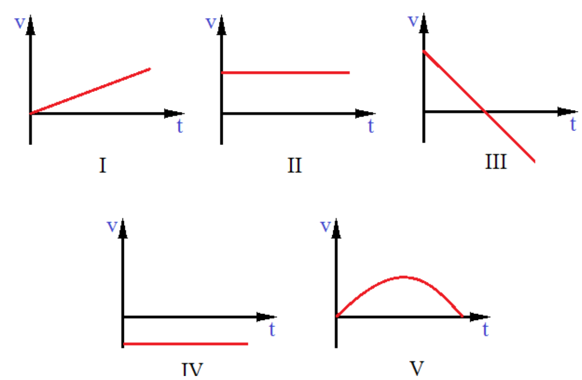


$$R = \frac{V^2 Sin(2\theta)}{g} \rightarrow Sin(2\theta) = \frac{Rg}{V^2} = \frac{(120m)(9.8m/s^2)}{(350m/s)^2} = 0.0096$$

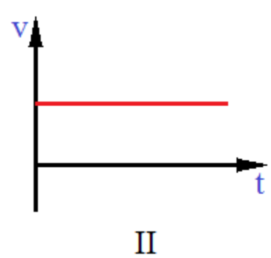
We take inverse sine on both sides to get:

$$2\theta = \sin^{-1}(0.0096) \rightarrow \theta = \mathbf{0.27^o}$$

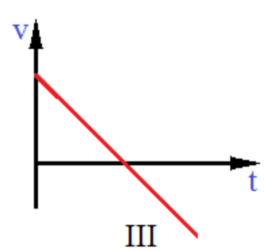
Problem 7.- A stone is thrown at an angle of 45° above the horizontal x-axis in the positive x-direction. If air resistance is ignored, which of the velocity versus time graphs shown below best represents V_x versus t and V_y versus t, respectively?



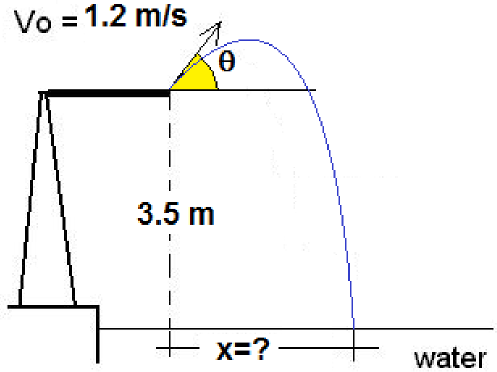
Solution: In the x-direction there is no acceleration, so the velocity is positive and constant:



In the y direction the velocity is decreasing from a positive value to a negative value, the slope being the value of “g”:



Problem 8.- A diver leaves the end of a 3.5 m-high diving board with an initial velocity of 1.2 m/s at an angle $\theta = 30^{\circ}$ with respect to the horizontal. Find the value of x.



Solution: The initial velocity can be decomposed in two components:

$v_{x1} = 1.2\cos 30^{\circ} = 1.04 m/s$

$v_{y1} = 1.2\sin 30^{\circ} = 0.60 m/s$

Next, we can use the equation $v_{y2}{}^2 = v_{y1}{}^2 + 2a_y y$ to find v_{y2} as follows:

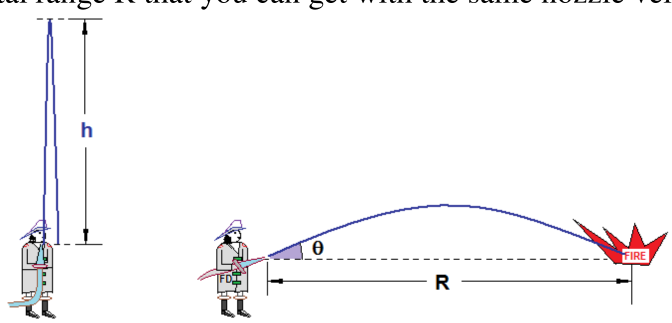
$v_{y2}{}^2 = 0.6^2 + 2(-9.8)(-3.5) \rightarrow v_{y2} = -8.3 m/s$

Then using the equation $v_{y2} = v_{y1} + a_y t$ we find the time:

$-8.3 = 0.6 + (-9.8)t \rightarrow t = 0.908s$

Finally, we can find x with: $x = v_{x1}t = (1.04)(0.908) =$ **0.94 m**

Problem 9.- The water from a fire hose pointing straight up reaches a height of h=35 m. What is the maximum horizontal range R that you can get with the same nozzle velocity?



Solution: The maximum height can be calculated by using the equation:

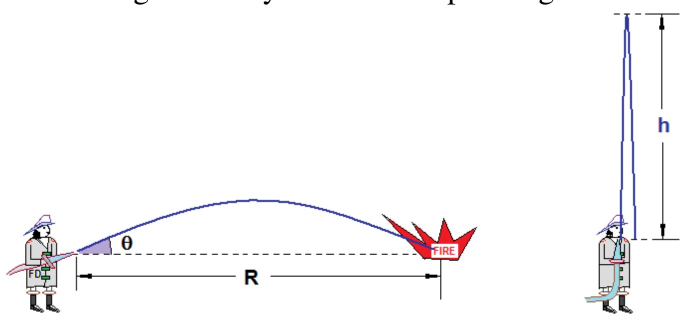
$$v_{y2}^2 = v_{y1}^2 + 2(-9.8)y \rightarrow y = \frac{v_{y1}^2}{2(9.8)}$$

But, in the horizontal direction, the maximum range happens when the angle is 45º, which means that:

$$R = \frac{v_1^2}{9.8}\sin(2\times 45^\circ) = \frac{v_1^2}{9.8}$$

This quantity is twice as much as the maximum height, so in this case the maximum horizontal range is **70m.**

Problem 9a.- A fire hose shoots water at 25 m/s and you can point the nozzle in any direction you want. Calculate the distance R that you reach horizontally shooting at $\theta = 25^\circ$ above the horizontal and the maximum height h that you can reach pointing the nozzle straight up.

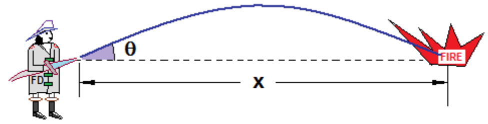


Solution:

a) $R = \dfrac{V_o^{\,2} Sin(2\theta)}{g} = \dfrac{25^2 \sin(2\times 25^\circ)}{9.8} = \mathbf{49\ m}$

b) $v_2^{\,2} = v_1^{\,2} + 2ax \rightarrow 0^2 = 25^2 + 2(-9.8)x \rightarrow x = \dfrac{25^2}{2\times 9.8} = \mathbf{32\ m}$

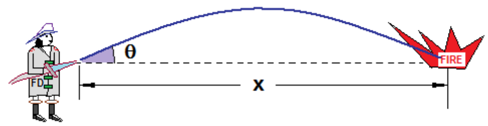
Problem 9b.- A fire hose shoots water at 25 m/s. Calculate the angle θ needed to reach a fire x=30 m away and at the same height as the nozzle. Neglect air resistance.



Solution: $R = \dfrac{v_o^{\,2}\sin(2\theta)}{9.8} \rightarrow 30 = \dfrac{25^2 \sin(2\theta)}{9.8} \rightarrow \sin(2\theta) = \dfrac{30\times 9.8}{25^2}$

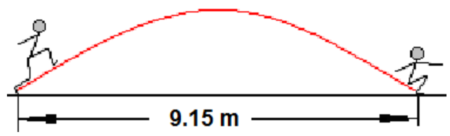
$$2\theta = \sin^{-1}\left(\frac{30\times 9.8}{25^2}\right) \rightarrow \theta = \frac{1}{2}\sin^{-1}\left(\frac{30\times 9.8}{25^2}\right) = \mathbf{14^o}$$

Problem 9c.- A fire hose shoots water at an angle $\theta = 30^\circ$. Calculate the speed needed to reach a fire located x=44 m away and at the same height as the nozzle. Neglect air resistance.



Solution: To get the range: $R = \frac{V^2 \sin(2\theta)}{g} \rightarrow V = \sqrt{\frac{Rg}{\sin(2\theta)}} = \sqrt{\frac{44m(9.8m/s^2)}{\sin(2\times30^\circ)}}$ = **22.3 m/s**

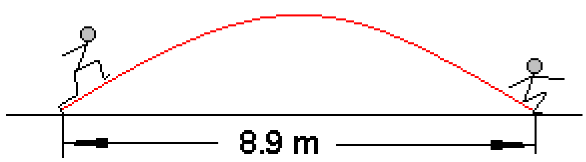
Problem 10.- According to witnesses, Carl Lewis had a long jump of 9.15m in 1982. Assuming it is true, estimate his initial speed considering a launch angle of 35°



Solution:

$$R = \frac{V_o^2 \sin(2\theta)}{g} \rightarrow V_o = \sqrt{\frac{Rg}{\sin(2\theta)}} = \sqrt{\frac{9.15\times9.8}{\sin(2\times35^\circ)}} = \mathbf{9.77\ m/s}$$

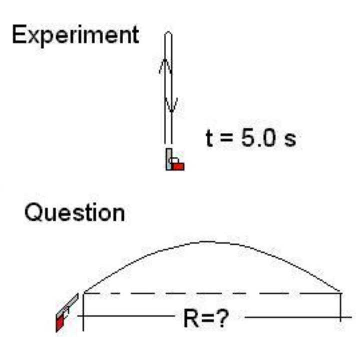
Problem 10a.- Bob Beamon's spectacular long jump of 8.90 m at the Mexico City Olympics of 1968 would last as a world record for 22 years. Estimate Bob Beamon's take off speed assuming he left the ground at a launch angle of 35.3°



Solution: If we approximate Bob Beamon's jump as a simple range problem we get:

$$8.9 = \frac{v_1^2}{9.8}\sin(2\theta) \rightarrow v_1 = \sqrt{\frac{8.9\times9.8}{\sin(2\theta)}} = \sqrt{\frac{8.9\times9.8}{\sin(2\times35.3^\circ)}} = \mathbf{9.62\ m/s}$$

Problem 11.- You want to find the maximum horizontal range (R) of a plastic dart gun that you just bought, so you do a little experiment to find the initial speed of the dart. You shoot the gun straight up and measure 5.0 seconds for the dart to land back at the barrel. How much is the maximum range of the gun?

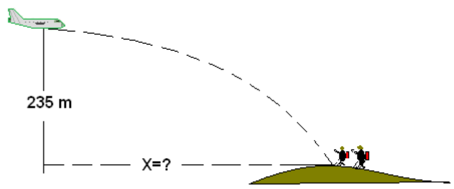


Solution: It takes 2.5 seconds to slow down to zero velocity, so the initial velocity is $v_{\circ} = 2.5 \times 9.8 = 24.5 m/s$

To find the maximum range we use the range equation with $\theta = 45^{\circ}$

$$R = \frac{v_{\circ}^{2} \sin(2\theta)}{9.8} = \frac{24.5^{2} \sin(2 \times 45)}{9.8} = \mathbf{61.25\ m}$$

Problem 12.- A rescue plane drops supplies to isolated mountain climbers on a rocky ridge 235m below. If the plane is traveling horizontally at 70 m/s, how far in advance (x) should the goods be dropped?

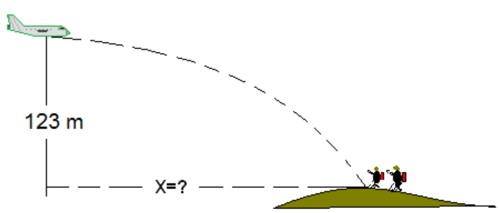


Solution: In the vertical direction the initial velocity is zero, the value of y is -235 and the acceleration is -9.8, so:

$$\mathrm{y} = \mathrm{v}_{1y}\mathrm{t} + \frac{1}{2}\mathrm{a}_{y}\mathrm{t}^{2} \rightarrow -235 = 0\mathrm{t} + \frac{1}{2}(-9.8)\mathrm{t}^{2} \rightarrow \mathrm{t} = \sqrt{\frac{2 \times 235}{9.8}} = 6.92s$$

And using the equation in the x-axis: $x = v_{x}t = 70 \times 6.92 =$ **485 m**

Problem 12a.- A rescue plane drops supplies to isolated mountain climbers on a rocky ridge 123m below. If the plane is traveling horizontally at 80 m/s, how far in advance (x) should the goods be dropped?



Solution: In the vertical direction the initial velocity is zero, the value of y is -123 and the acceleration is -9.8, so:

$$\mathrm{y} = \mathrm{v}_{1y}\mathrm{t} + \frac{1}{2}\mathrm{a}_{y}\mathrm{t}^{2} \rightarrow -123 = 0\mathrm{t} + \frac{1}{2}(-9.8)\mathrm{t}^{2} \rightarrow \mathrm{t} = \sqrt{\frac{2 \times 123}{9.8}} = 5s$$

And using the equation in the x-axis: $x = v_{x}t = 80 \times 5 =$ **400 m**

Problem 13.- Punt: A football left the punter's foot at a height of 1.2 m above the ground with an initial velocity of 20m/s at an angle of 37º above the horizontal. How far does the football travel before hitting the ground?

Solution: In the vertical direction we have an initial velocity of $v_{y1} = 20\sin(37^\circ) = 12$, a final value of y=-1.2 and we also know the acceleration $a_y = -9.8$, so we can find the time by writing the equation:

$$y = v_{1y}t + \frac{1}{2}a_y t^2 \rightarrow -1.2 = 12t + \frac{1}{2}(-9.8)t^2 \rightarrow 4.9t^2 - 12t - 1.2 = 0$$

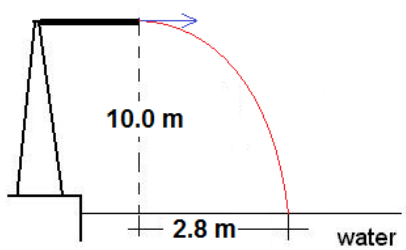
And now we solve the quadratic equation:

$$t = \frac{12 \pm \sqrt{12^2 + 4\times 4.9\times 1.2}}{2\times 4.9} = 2.54s \quad or \quad -0.09s$$

We take the positive solution and find x:

$$x = v_x t = (20\cos 37^\circ)\times 2.54 = \mathbf{41m}$$

Problem 14.- A diver pushes horizontally off a platform 10.0m above the water. If the diver hits the surface 2.8 m beyond the platform, how much was his initial velocity?



Solution: In the vertical direction the initial velocity is zero, the value of y is -10 and the acceleration is -9.8, so:

$$\mathrm{y} = \mathrm{v}_{1y}\mathrm{t} + \frac{1}{2}\mathrm{a}_y\mathrm{t}^2 \rightarrow -10 = 0\mathrm{t} + \frac{1}{2}(-9.8)\mathrm{t}^2 \rightarrow \mathrm{t} = \sqrt{\frac{2\times 10}{9.8}} = 1.43s$$

And using the equation in the x-axis: $x = v_x t \rightarrow v_x = \frac{x}{t} = \frac{2.8}{1.43} = \mathbf{2m/s}$

Problem 15.- Two golfers each hit a ball with the same speed, but one at 25º with the horizontal and the other at 65º. Which ball goes further? Which one hits the ground first?

Ignore air resistance

Solution: Since the angles are complementary (they add to 90 degrees) the **range is the same,** however the ball hit at **25º will hit the ground first** because its velocity has a smaller vertical component.