Physics I

Velocity Basics

Definition of velocity in one dimension: $v = \frac{x}{t}$

1 mile = 1609 m

Problem 1.- Donovan Bailey of Canada broke the world record in the 100m dash at the 1996 Atlanta Olympics with a time of 9.84 seconds. With this gold medal, he became "the world's fastest man".

a. What was his average velocity over this distance in miles per hour?

b. A couple of days later, Michael Johnson set the world record in the 200m dash with a time of 19.32 seconds. What was his average speed in miles per hour?

c. Was Donovan Bailey the world's fastest man?

Solution: a. Donovan Bailey’s average velocity is:

$$\bar{v} = \frac{100\text{m}}{9.84\text{s}}\left(\frac{3600\text{s}}{1\text{h}}\right)\left(\frac{1\text{mile}}{1609\text{m}}\right) = \mathbf{22.7\ mph}$$

b. Michael Johnson’s average speed:

$$\bar{v} = \frac{200\text{m}}{19.32\text{s}}\left(\frac{3600\text{s}}{1\text{h}}\right)\left(\frac{1\text{mile}}{1609\text{m}}\right) = \mathbf{23.2\ mph}$$

c. Michael Johnson was faster than Donovan Bailey!

Problem 2.- In the Olympic Games in Seoul, Korea, Carl Lewis of the USA ran 100.0 meters in 9.92 seconds to win the gold medal. Find his average velocity in miles per hour. Use the correct number of significant figures.

Solution: By definition of velocity in one dimension:

$$v = \frac{x}{t} = \frac{100.0\text{m}}{9.92\text{s}} = 10.08\text{m/s}$$

In miles per hour: $v = 10.08\text{m/s}\left(\frac{3600\text{s}}{1\text{h}}\right)\left(\frac{1\text{mile}}{1609\text{m}}\right) = \mathbf{22.6\ miles/hour}$

Problem 3.- Usain Bolt of Jamaica broke the world record in the 100m dash in 2009 with a time of 9.58 seconds.

a. What was his average speed over this distance in miles per hour?

b. He also broke the world record in the 200m dash with a time of 19.19 seconds. What was his average speed in miles per hour?

c. In which race was his average speed higher?

Solution: Usain Bolt's average speed in the 100 m race was:

$$\overline{v} = \frac{100\text{m}}{9.58\text{s}}\left(\frac{3600\text{s}}{1\text{h}}\right)\left(\frac{1\text{mile}}{1609\text{m}}\right) = \mathbf{23.36\ mph}$$

And his average speed in the 200 m race was

$$\overline{v} = \frac{200\text{m}}{19.19\text{s}}\left(\frac{3600\text{s}}{1\text{h}}\right)\left(\frac{1\text{mile}}{1609\text{m}}\right) = \mathbf{23.32\ mph}$$

So, he was faster in the 100 m race.

Note: If we had asked for the average velocity instead of average speed, the result for the 200 m race would have been considerably less due to the curve in the first 100 meters.

Problem 4.- The cheetah is the fastest animal on land. Calculate how long (in seconds) it would take a cheetah to run 460 m at its maximum speed of 70 miles per hour.

Solution: First, we convert the velocity from miles per hour to m/s:

$$v = 70\frac{\text{mile}}{\text{hour}}\left(\frac{1\text{hour}}{3600\text{s}}\right)\left(\frac{1609\text{m}}{1\text{mile}}\right) = 31.28\frac{\text{m}}{\text{s}}$$

And now, the time will be given by:

$$t = \frac{460m}{31.28m/s} = 14.7\text{s}$$

You can approximate this to 15s.

Problem 5.- Find the equation for the time of flight of an airplane back and forth a distance L if the velocity of the plane in air is v and the wind velocity is v' in two cases:

i) Wind parallel to the direction of the plane.

ii) Wind perpendicular to the direction of the plane.

Solution:

i) In the first case, the speed with respect to the terrain will be v+v' on the outbound leg and v-v' on the inbound leg, giving a total time:

$$t = \frac{L}{v+v'} + \frac{L}{v-v'} = \frac{2vL}{v^2 - v'^2}$$

Another way to express this time is:

$$t = \frac{2L}{v}\left(\frac{1}{1 - v'^2/v^2}\right)$$

In this equation it is explicitly shown that the time for the round trip is longer than if the velocity were constant v.

ii) In the second case, the speed with respect to the terrain will be $\sqrt{v^2 - v'^2}$ in both directions, so the time will be:

$$t = \frac{2L}{\sqrt{v^2 - v'^2}}$$

We can also express this time as:

$$t = \frac{2L}{v}\left(\frac{1}{\sqrt{1 - v'^2/v^2}}\right)$$

Once again we notice that the time is longer than if the velocity were constant v, and this is even slower than the first case.

Problem 6.- In a new "crazy-triathlon" competition, you have to run sideways for 1km, then backwards for 1km and finally 1km any way you want (free-style).

Competitor A has speeds of 4m/s, 6m/s, and 8m/s for the three legs of the competition.

Competitor B has speeds of 5m/s, 6m/s, and 7m/s.

Which competitor wins the race?
Do you get any insight about average speeds from this problem?

Solution: You can think of this problem as analogous to the swimming medley with different stroke styles. To find the total time for a race we need to consider each leg of the competition individually and add the times.

For competitor A:

$$T_A = \frac{1000\text{m}}{4\text{m/s}} + \frac{1000\text{m}}{6\text{m/s}} + \frac{1000\text{m}}{8\text{m/s}} = 542\text{s}$$

For competitor B:

$$T_B = \frac{1000\text{m}}{5\text{m/s}} + \frac{1000\text{m}}{6\text{m/s}} + \frac{1000\text{m}}{7\text{m/s}} = 510\text{s}$$

Competitor B uses less time and wins.

This is yet another example where the average speed cannot be calculated as a simple arithmetic average. Notice that 4, 6 and 8 give an arithmetic average of 6, same as 5, 6 and 7, but the average speed is higher for B than for A and less than 6m/s in both cases:

$$V_A = \frac{3000\text{m}}{542\text{s}} = 5.53\text{m/s}$$
$$V_B = \frac{3000\text{m}}{510\text{s}} = 5.89\text{m/s}$$

When the distances are the same, the average velocity needs to be the "harmonic average", not the arithmetic average.