Physics I

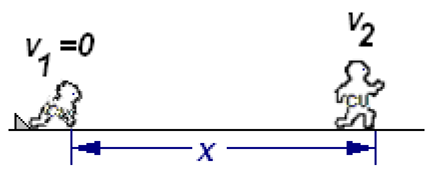
Constant Acceleration

Equations for constant acceleration in 1 dimension

$$x = v_1 t + \frac{1}{2} a t^2 \qquad v_2 = v_1 + at \qquad {v_2}^2 = {v_1}^2 + 2ax \qquad \langle v \rangle = \frac{v_1 + v_2}{2} = \frac{x}{t}$$

Problem 1.- In the 100-m race an athlete accelerates uniformly from rest to his top speed of v_2=10m/s in the first x=15m as shown in the figure.

a) Find his acceleration in the first 15m.
b) Find the time it takes to cover those first 15 m.



Solution:

a) The acceleration in those first 15m can be calculated using:

$$v_2^2 = v_1^2 + 2ax \rightarrow a = \frac{v_2^2 - v_1^2}{2x}, \text{ so: } a = \frac{10^2 - 0^2}{2(15)} = \mathbf{3.33 m/s^2}$$

b) The time it takes him to run those first 15m: Since $x = v_1 t + \frac{1}{2} a t^2$, we have:

$$15 = 0t + \frac{1}{2}(3.33)t^2 \rightarrow t = \sqrt{\frac{2 \times 15}{3.33}} = \mathbf{3\ s}$$

Problem 2.- If we could neglect air resistance and other secondary effects, how long would it take for a bullet fired straight up with an initial velocity of 350m/s to hit the shooter?

Solution: We can use the equation: $y = v_{1y} t + \frac{1}{2} a_y t^2$ to solve the problem. Notice that y=0 because the bullet comes back to the same place, so:

$$0 = (350)t + \frac{1}{2}(-9.8)t^2$$

$$\rightarrow t = 0 \quad or \quad t = \frac{350}{4.9} = \mathbf{71s}$$

Problem 3.- You dangle your watch from a thin piece of string while the jetliner you are in accelerates for takeoff. Calculate the acceleration if the string makes an angle of 35º with respect to the vertical.

Solution: We know that the force on a string, cable, thin wire, thread or chain is along the same direction (it cannot be sideways). Now, on the watch there are two forces acting: its own weight and the string force.

The watch is in equilibrium vertically, so the weight has to cancel the vertical component of the string force vector:

$$F_{string} \cos 35^\circ = mg$$

In the horizontal direction, there is only one force, namely the horizontal component of the string tension and using Newton's second law:

$$-F_{string} \sin 35^\circ = ma$$

Dividing the second equation by the first, we get:

$$\frac{-F_{string} \sin 35^\circ}{F_{string} \cos 35^\circ} = \frac{ma}{mg} \rightarrow -\frac{\sin 35^\circ}{\cos 35^\circ} = \frac{a}{g} \rightarrow a = -g \tan 35^\circ = \mathbf{-6.86 m/s^2}$$

Problem 4.- A rocket needs to reach a speed of 8,000 m/s starting from rest.
Assuming a constant acceleration of 25 m/s^2 calculate:

i) The time it will take to reach that velocity
ii) The distance covered in that time

Solution:

i) To find the time we use the equation: $v_2 = v_1 + at$

$$8000 = 0 + 25t \rightarrow t = \frac{8000}{25} = \mathbf{320\ s}$$

ii) To find the distance we can use the fact that the average velocity is 4000 m/s and then:

$$x = \langle v \rangle t = 4000 \times 320 = \mathbf{1,280,000\ m}$$

Problem 5.- A plane on an aircraft carrier has only 122m to accelerate on take-off. How much must be the acceleration (assumed constant) if it has to reach 195 km/h starting from rest?

Solution: First, let us calculate the final velocity in m/s:

$$v_2 = 195 \frac{km}{h} \left(\frac{1h}{3600s} \right) \left(\frac{1000m}{1km} \right) = \mathbf{54.2 m/s}$$

Now we use this to find the acceleration:

$$v_2{}^2 = v_1{}^2 + 2ax \rightarrow a = \frac{v_2{}^2 - v_1{}^2}{2x} = \frac{54.2^2 - 0^2}{2\times 122} = \mathbf{12\ m/s^2}$$

Problem 6.- A car traveling at 88 km/h strikes a tree. Thanks to the seat belts, air bag and modern design of the front of the car, the driver is brought to rest with constant acceleration after traveling 1.1 m. What was the driver's acceleration during the collision?

Solution: $v_1 = 88\frac{km}{h}\left(\frac{1h}{3600s}\right)\left(\frac{1000m}{1km}\right) = 24.4$ m/s

The final velocity is zero and the acceleration is:

$$a = \frac{0-(24.4m/s)^2}{2(1.1m)} = \mathbf{272\ m/s^2}$$

Problem 7.- A driver going at 60 miles/hour (=26.8m/s) sees a deer crossing the road in front of him and hits the brakes when he is 60 m away. The coefficient of static friction is 0.75 (the car has ABS, so it doesn't slip). Is the deer safe?

Solution: We can calculate the distance that the driver needs to stop: Its initial velocity is 26.8m/s and since $\mu_K = 0.75$ it means that its acceleration will be -0.75(9.8m/s^2) so the distance needed is:

$$V_2{}^2 = V_1{}^2 + 2ax \rightarrow 0 = V_1{}^2 + 2ax = (26.8\text{m/s})^2 + 2(-7.3m/s^2)x$$

$$x = \frac{(26.8\text{m/s})^2}{2(7.3m/s^2)} = \mathbf{49.2\ m}$$

Since this is less than the distance to the deer, it gets to live another day.

Problem 8.- Calculate the distance covered by a "model T" car that accelerates from zero to its maximum speed of 45 mph in 15.0 seconds. [1 mile=1609m]

Solution: Converting 45 mph to meters/second:

$$v_2 = \frac{45\text{miles}}{\text{h}}\left(\frac{1\text{h}}{3600\text{s}}\right)\left(\frac{1609\text{m}}{1\text{mile}}\right) = 20.1\text{m/s}$$

The average velocity is:

$$\bar{v} = \frac{v_1 + v_2}{2} = \frac{0 + 20.1\text{m/s}}{2} = 10.05\text{m/s}$$

And the distance covered in 15s:

$$x = \bar{v}t = (10.05\text{m/s})(15\text{s}) = \mathbf{152\ m}$$

Problem 9.- A roadster accelerates from zero to 105 miles per hour in 6.0 seconds. Calculate the average acceleration in m/s^2. (1 mile = 1609 m)

Solution: First, let us convert the final velocity to m/s:

$$V_2 = 105\frac{\text{mile}}{\text{h}}\left(\frac{1609\text{m}}{1\text{mile}}\right)\left(\frac{1\text{h}}{3600\text{s}}\right) = 46.93 \text{ m/s}$$

The initial velocity is zero, and so, the definition of acceleration gives us:

$$a = \frac{V_2 - V_1}{t} = \frac{46.93\text{m/s} - 0}{6.0\text{s}} = \mathbf{7.8\ m/s^2}$$

Problem 10.- A plane accelerates along a runway at $6.55 m/s^2$ staring from rest. It needs to reach 370 km/h for take-off. What should be the minimum length of the runway for a safe take-off?

Solution: If the plane starts from rest, $v_1=0$ and the required velocity for takeoff is:

$$v_2 = 370\frac{\text{km}}{\text{h}}\left(\frac{1\text{h}}{3600\text{s}}\right)\left(\frac{1000\text{m}}{1\text{km}}\right) = 102.8\text{m/s}$$

To get the length of the runway we use the formula $v_2{}^2 = v_1^2 + 2ax$:

$$(102.8\text{m/s})^2 = (0)^2 + 2(6.55\text{m/s}^2)x \rightarrow x = \frac{(102.8\text{m/s})^2}{2(6.55\text{m/s}^2)} = \mathbf{807\ m}$$

Problem 11.- In coming to a stop, a car leaves a 95m-long skid mark along the highway. Assuming an acceleration of $-7.5 m/s^2$, determine the initial velocity of the car just before the brakes were applied (you can give the answer in m/s).

Solution: In this problem we know the final velocity ($V_2=0$) the distance traveled (x=95m) and the acceleration ($a=-7.5 m/s^2$). The easiest way to find the initial velocity is using the equation:

$$V_2{}^2 = V_1{}^2 + 2ax$$

With the information of the problem:

$$(0)^2 = V_1{}^2 + 2(-7.5\text{m/s}^2)(95\text{m}) \rightarrow V_1{}^2 = 1425\text{m}^2/\text{s}^2$$

We only take the positive solution in this case, which is: $\mathbf{V_1 = 38\ m/s}$

Problem 12.- The brakes of an 855-kg car apply a force of -4880N. Calculate the distance needed to stop the car if it is going at 65 miles per hour. [1 mile=1609 m]

Solution: The speed of the car in m/s is: $v = 65\frac{\text{mile}}{\text{h}}\left(\frac{1\text{h}}{3600\text{s}}\right)\left(\frac{1609\text{m}}{1\text{mile}}\right) = 29.1\text{m/s}$

To stop the car the kinetic energy is totally lost. The work done by the braking force will be equal to the loss:

$$F_{braking}d = \frac{1}{2}mv^2 \rightarrow d = \frac{\frac{1}{2}mv^2}{F_{braking}} = \frac{mv^2}{2F_{braking}} = \frac{655kg(29.1m/s)^2}{2(3880N)} = \textbf{71.5 m}$$

Problem 13.- You design the front of a car so in the case of a collision at 55 km/h the passenger will experience a maximum of 20 "g"s of acceleration.
Calculate how much distance you have to slow down the passenger to rest without exceeding the maximum acceleration.
1 "g"=9.8m/s^2

Solution:

$$v_2 = 55\frac{km}{h}\left(\frac{1h}{3600}\right)\left(\frac{1000m}{1km}\right) = 15.3m/s$$

$$v_2{}^2 = v_1{}^2 + 2ax \rightarrow 0^2 = 15.3^2 + 2(20\times 9.8)x \rightarrow x = \frac{15.3^2}{2\times 20\times 9.8} = \textbf{0.6 m}$$