# Physics Courseware <br> Physics I <br> Center of Mass 

Center of mass: $\quad X_{C M}=\frac{\sum m_{i} x_{i}}{\sum m_{i}} \quad$ and $\quad Y_{C M}=\frac{\sum m_{i} y_{i}}{\sum m_{i}}$
Or $\quad X_{C M}=\frac{\int x d m}{\int m} \quad$ and $\quad Y_{C M}=\frac{\int y d m}{\int m}$
Problem 1.- Find the center of mass $X_{C M}$ and $Y_{C M}$ of the following object made with three masses connected with thin rods.
Ignore the mass of the rods and take each square in the grid as 1 m .


Problem 1a.- Find the center of mass $X_{C M}$ and $Y_{C M}$ of the following object made with three masses connected with thin rods.
Ignore the mass of the rods and take each square in the grid as 1 m .


Problem 2.- Find the center of mass of a wire with the shape shown in the figure.


Problem 3.- Find the center of mass of the plate shown in the figure, knowing that its thickness and density are uniform. It is a square of side $\mathrm{L}=2.00$ with a circle cut out located at $(1.50,1.50)$ with a radius $\mathrm{R}=0.220$ as shown in the figure. Give the answer with 3 significant figures.


Problem 4.- Calculate the location of the center of mass of the water molecule if the mass of hydrogen is 1 u and the mass of oxygen is 16 u .


Problem 5.- Calculate the location of the center of mass of the solid cone shown in the figure. Assume it has uniform density, radius at the base $\mathrm{R}=0.2 \mathrm{~m}$ and height $\mathrm{H}=0.8 \mathrm{~m}$
Suggestion: You can integrate thin disks of radius " $r$ " to get the center of mass. Notice that the radius would be $r=R(1-z / H)$


Problem 5a: Calculate the location of the center of mass of the solid cone shown in the figure. Assume it has uniform density, radius at the base $\mathrm{R}=0.2 \mathrm{~m}$ and height $\mathrm{H}=0.8 \mathrm{~m}$ There is a cylinder carved out at the base of radius $r=0.1 \mathrm{~m}$ and height $\mathrm{h}=0.2 \mathrm{~m}$


Problem 6.- Find the center of mass ( $\mathrm{x}_{\mathrm{cm}}$ and усм ) of the following plate that has uniform thickness:


Problem 7.- Calculate the location of the center of mass of the HCl molecule if the mass of hydrogen is 1 u and the mass of chlorine is 35 u and the distance between the atoms is $1.3 \AA$. In your answer, specify where the center of mass is with respect to the chlorine nucleus.

Problem 8.- How far from the center of the Earth is the center of mass of the Earth-Moon system?
Consider the distance between the center of the Earth and the center of the Moon to be $384 \times 10^{3}$ km , the mass of the Earth $=5.98 \times 10^{24} \mathrm{~kg}$ and the mass of the Moon $7.35 \times 10^{22} \mathrm{~kg}$.

Problem 9.- A metallic plate with constant thickness and density has the shape of a square with a side of 2 m . A round disk of radius 0.5 m is removed from the square leaving the shape shown in the figure. Calculate where the center of mass is located.


Problem 10.- Find how far from Pluto is the center of mass of the Pluto-Charon system. The mass of Pluto is $1.30 \times 10^{22} \mathrm{~kg}$, the mass of Charon is $1.51 \times 10^{21} \mathrm{~kg}$ and the distance between them is $19,600 \mathrm{~km}$.
Given that the radius of Pluto is $1,153 \mathrm{~km}$, is the Center of Mass inside or outside Pluto?

Problem 11.- Find the center of mass of the following plate that has uniform thickness and density:


Problem 12.- Seven particles of mass 1 kg each are located at the corners of an octagon as shown in the figure. Calculate the position of the center of mass ( $\mathrm{x}_{\mathrm{CM}}$ ).


Problem 12a: A system of nine particles of mass 1 kg each are located at the corners of an octagon, plus one particle at the $(6,0)$ position as shown in the figure. Calculate the location of the center of mass of the system $\left(X_{C M}\right)$.


Problem 13.- A solid rocket nose has a shape that can be modeled as a cylinder with variable radius $r=2 \sqrt{x}$ as shown in the figure. Calculate the center of mass of the nose.
[Suggestion: divide the nose in infinitesimally thin disks of area $\pi r^{2}$ ]


Problem 14.- Find the center of mass of the following plate that has uniform thickness and density. Take the center of the large circle to be the origin.


Problem 15.- Find the center of mass of the following plate that has uniform thickness and density:


