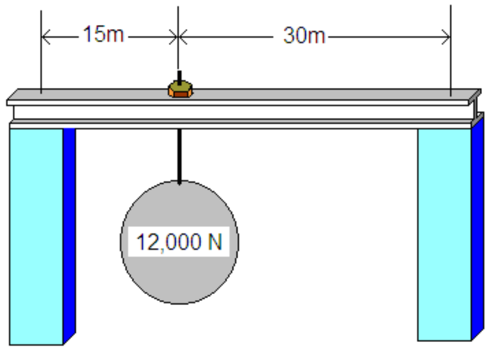
Physics I

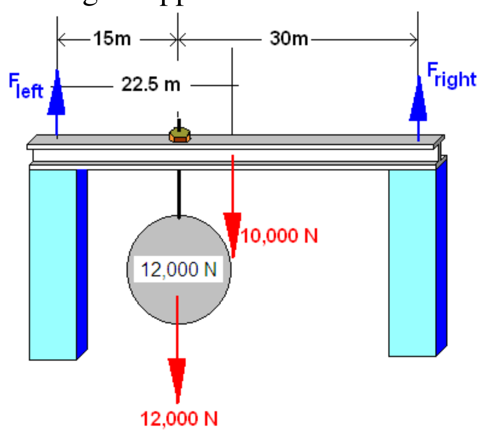
Statics Problems, Beams

Equilibrium equations $\sum F = 0$ and $\sum \tau = 0$ Where $\tau = Fr\sin\angle_F^r$

Problem 1.- The weight of a beam is 10,000 N and it is supporting a load of 12,000 N as shown in the figure. Calculate the reaction forces supplied at the supports on both ends:



Solution: There are four forces acting on the beam: The weight of the beam, the weight of the load, the left support force and the right support force as shown below:



Newton's second law says:

$$\sum F_y = 0 \rightarrow F_{left} + F_{right} = 10{,}000N + 12{,}000N = 22{,}000N$$

In addition, an equation for the torque looks as follows: (using the left support as the center of rotation)

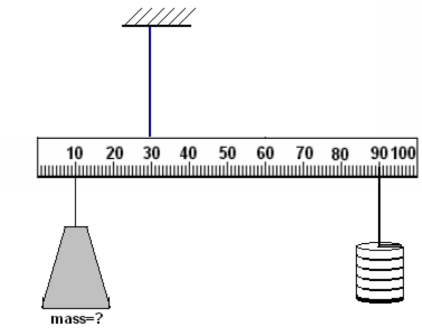
$$\sum \tau = 0 \rightarrow 10{,}000N \times 22.5m + 12{,}000N \times 15m = F_{right} \times 45m$$

This equation gives us $F_{right} = \dfrac{10{,}000N \times 22.5m + 12{,}000N \times 15m}{45m} = \mathbf{9{,}000N}$

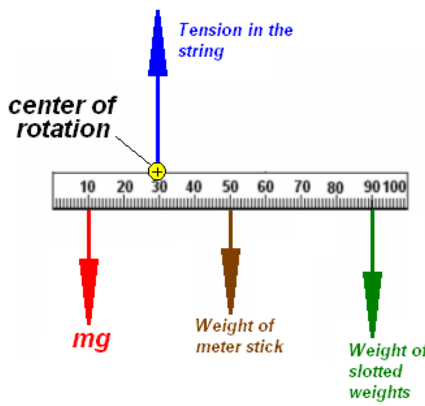
And replacing this in the first equation:

$$F_{left} + 9{,}000N = 22{,}000N \rightarrow F_{left} = \mathbf{13{,}000N}$$

Problem 2.- The following arrangement of meter stick, unknown object and slotted weights is in equilibrium hanging from a string. Calculate the mass of the unknown object if the mass of the meter stick is 0.035 kg (with its center of mass at the 50cm mark) and the mass of the slotted weights is 0.150 kg.



Solution:



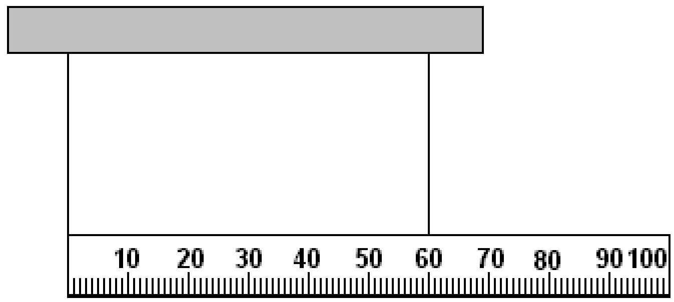
The figure shows a free body diagram of the meter stick with the four forces that are acting on it. The conditions of equilibrium tell us that the sum of torques must be zero. We choose the center of rotation at the 30cm mark because that eliminates the tension in the string from the equation, giving instead:

$$mg(20cm) - (0.035kg)g(20cm) - (0.150kg)g(60cm) = 0$$

where the counterclockwise direction has been considered positive.
And solving for the unknown mass we get:

$$m = \frac{(0.035kg)(20cm) + (0.150kg)(60cm)}{20cm} = \mathbf{0.48\ kg}$$

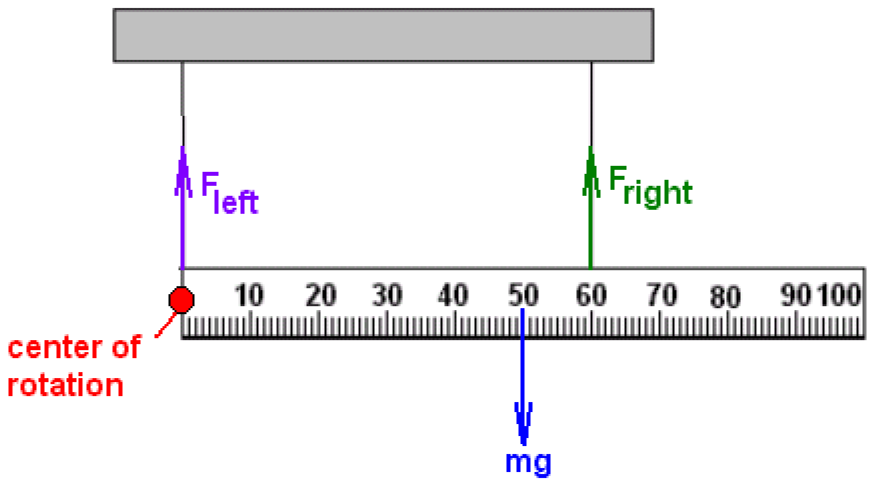
Problem 3.- The mass of the meter stick is 0.035 kg. Calculate the force on each supporting string if the center of mass is at the 50cm mark.



Solution: The weight of the meter stick is (0.035 kg)(9.8m/s^2) = 0.343 N
The sum of the two support forces is equal to the weight:

$$F_{left} + F_{right} = 0.343\text{N}$$

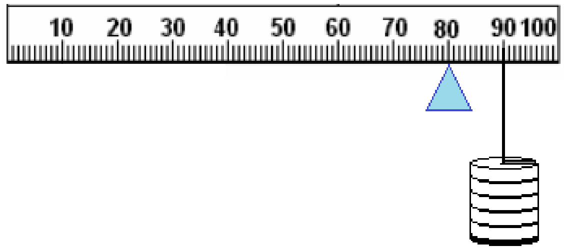
Also, since the meter stick is not rotating, the sum of torques is zero. For example, using the left support as center of rotation:



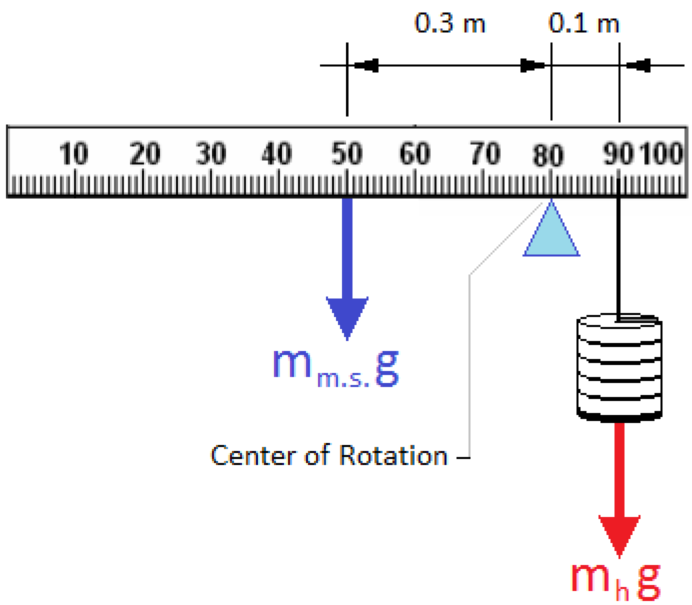
$$\text{mg}(50\text{cm}) - F_{right}(60\text{cm}) = 0 \rightarrow F_{right} = \frac{5}{6}\text{mg} = \mathbf{0.286\ N}$$

The other force is then 0.343N-0.285N = **0.057 N**

Problem 4.- The meter stick is in equilibrium as shown in the figure.
Calculate the mass of the meter stick knowing that its center of mass is at the 50cm mark, and the slotted weights with the hanger have a mass of 90g.



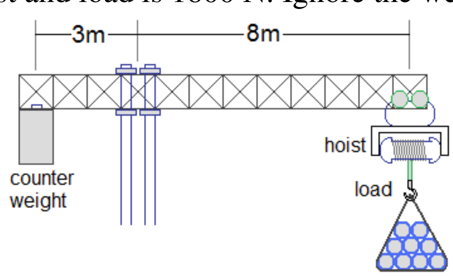
Solution: Consider the center of rotation at the pivot point, the mass of the meter stick to be $m_{m.s.}$ and the mass of the hanger (with the slotted weights) to be m_h as shown in the figure



Then, to be in equilibrium the sum of the torques should be zero, so

$$m_{m.s.}g \times 0.3 - m_h g \times 0.1 = 0 \rightarrow m_{m.s.} = \frac{m_h}{3} = \mathbf{30\ g}$$

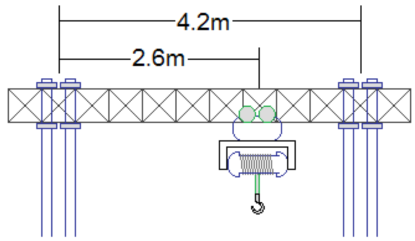
Problem 5.- The crane in the figure is in equilibrium (sum of forces is zero and sum of torques also zero). Find the counterweight on the left side of the crane and the vertical force on the support. The weight of the hoist and load is 1800 N. Ignore the weight of the beam.



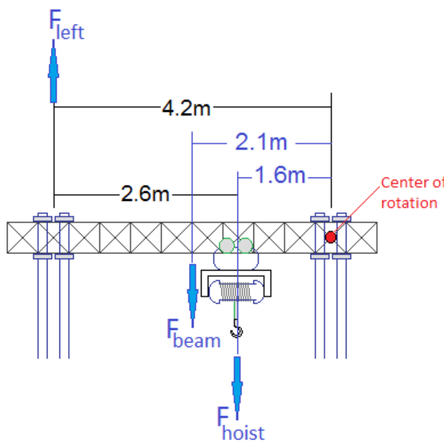
Solution: $W_{counter-weight} \times 3 = 1800 \times 8 \rightarrow W_{counter-weight} = \mathbf{4{,}800\ N}$

$F_{\text{support}} \times 3 = 1800 \times 11 \rightarrow F_{\text{support}} = \mathbf{6{,}600\ N}$

Problem 6.- Find the vertical force on the left support of the crane shown in the figure. The weight of the hoist is 2,100 N and the weight of the beam is 3,500 N.



Solution: We can use the second condition of equilibrium to solve the problem, that is $\sum \tau = 0$ and we choose the right support as the center of rotation because that will eliminate that force from the equation. Notice that in the figure we calculate the arm of each force with respect to the center of rotation.



The equation will then be $\sum \tau = 0 \rightarrow -F_{left} \times 4.2 + 3500 \times 2.1 + 2100 \times 1.6 = 0$

So, the solution is $F_{left} = \mathbf{2{,}550\ N}$