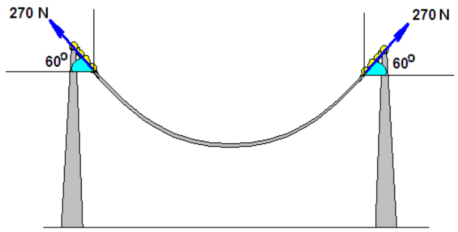
Physics I

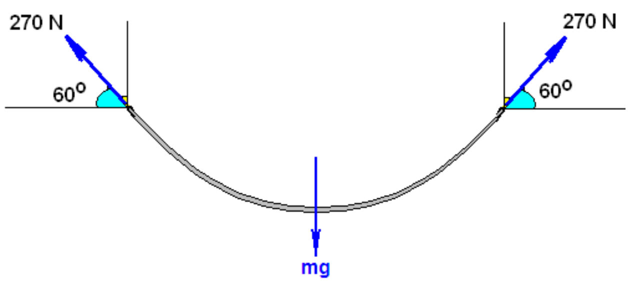
Statics Problems Cables

Elasticity: $\Delta L = \dfrac{FL}{EA}$ (the "flea" equation)

Problem 1.- Calculate the mass of the cable if the forces on the supports are as indicated:



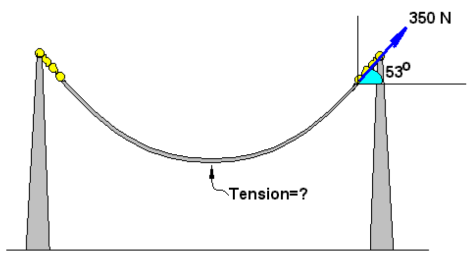
Solution: A free body diagram of the cable shows there are only three forces acting on it:



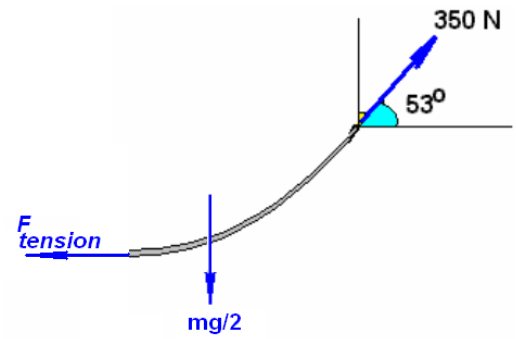
The sum of the forces in the vertical direction has to be zero for the cable to be in equilibrium, so:

$$2\times 270\text{N}\times\sin 60^\circ = mg \rightarrow m = \frac{2\times 270\text{N}\times\sin 60^\circ}{9.8m/s^2} = \mathbf{48\ kg}$$

Problem 2.- Calculate the tension at the center of the cable if the force on the right support is 350N:



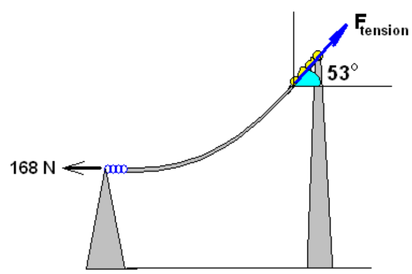
Solution: A free body diagram of *half* the cable shows that there are only three forces acting on it:



The sum of the forces in the horizontal direction must be zero for the cable to be in equilibrium, so:

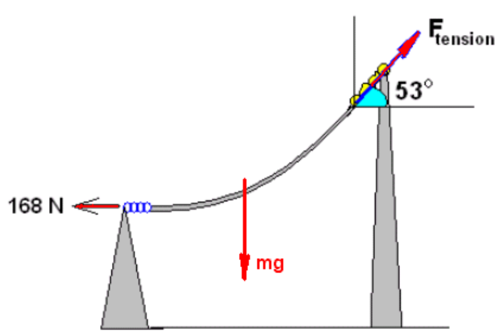
$350N\cos(53^{\circ}) - F_{tension} = 0 \rightarrow F_{tension} = 350N\cos(53^{\circ}) =$ **210 N**

Problem 3.- Calculate the force of tension at the right support of the cable if the force on the left support is 168N, horizontal, as shown:



Solution (A) By decomposition in components.

You have three forces action on the cable:



$F_{tension} = (F\cos 53^{\circ}, F\sin 53^{\circ})$

$F_{left} = (-168, 0)$

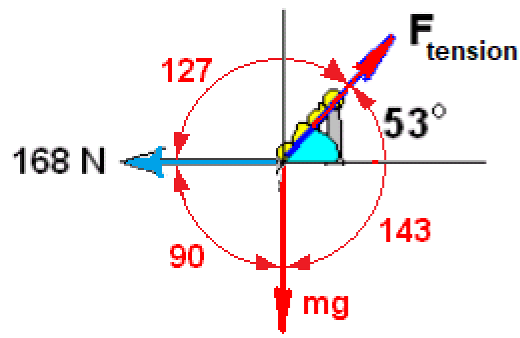
$\text{Weight} = (0, -mg)$

For the cable to be in equilibrium the sum of the forces must be zero, so:

$$\sum F_x = 0 \rightarrow F\cos 53^\circ - 168 = 0 \rightarrow F = \frac{168}{\cos 53^\circ} = \mathbf{280\ N}$$

Solution (B) By law of sines.

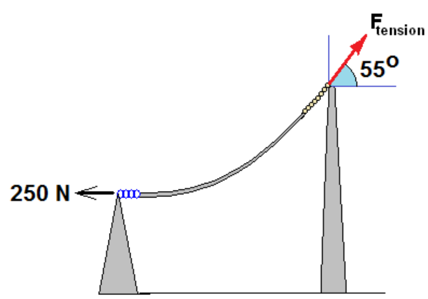
The three forces acting on the cable must form a triangle, so we can write the law of sines with the angles shown in red in the figure:



The equation is:

$$\frac{168}{\sin 143^\circ} = \frac{F}{\sin 90^\circ} \rightarrow F = \frac{168 \sin 90^\circ}{\sin 143^\circ} = \mathbf{280\ N}$$

Problem 3a.- Calculate the mass of the cable shown in the figure if the tension at the right support makes an angle of 55° to the horizontal and the force on the left support is 250N, horizontal.



Solution: The horizontal component of the force $\mathbf{F_{tension}}$ must be equal to 250N, so

$$F_{tension} \cos 55^\circ = 250N \rightarrow F_{tension} = 436N$$

And the vertical component of the force $\mathbf{F_{tension}}$ must be equal to the weight of the cable, so:

$$F_{tension} \sin 55^\circ = mg \rightarrow m = \frac{436 \sin 55^\circ}{9.8} = \mathbf{36\ kg}$$

Problem 4.- What would be the maximum load that you can lift with a single steel cable that has an effective cross section of 1 in^2 if you want a safety factor of 5?

(1 inch = 0.0254 m)

Solution: The breaking point happens when:

$$F = 1inch^2 \times 500\times10^6 N/m^2 = (0.0254m)^2 \times 500\times10^6 N/m^2 = 322{,}580\text{N}$$

but with a safety factor of 5 we only will lift a load of **64,500 N,** which corresponds to a mass of 6.5 tons.

Problem 5.-

a) What is the minimum cross-sectional area of a steel cable from which is suspended a 450kg load. Use a safety factor of 7. [Ultimate strength of steel = 500×10^6 N/m^2]

b) If the cable is 12m long, how much does it elongate?
[Young's modulus of steel = 200×10^9 N/m^2]

Solution:
a) The breaking point will happen when the stress is equal to the ultimate strength:

$$\frac{F}{A} = 500\times10^6 N/m^2 \rightarrow A = \frac{F}{500\times10^6 N/m^2} = \frac{450kg(9.8m/s^2)}{500\times10^6 N/m^2} = 8.8\times10^{-6}\text{m}^2$$

But we multiply this by 7 (the safety factor) A = $7\times8.8\times10^{-6}$m^2 = **62×10^{-6}m^2**

b) To get the elongation we use the "flea" equation:

$$\Delta L = \frac{FL}{EA} = \frac{450kg(9.8m/s^2)(12m)}{(200\times10^9 N/m^2)(62\times10^{-6}m^2)} = \textbf{0.0043 m}$$