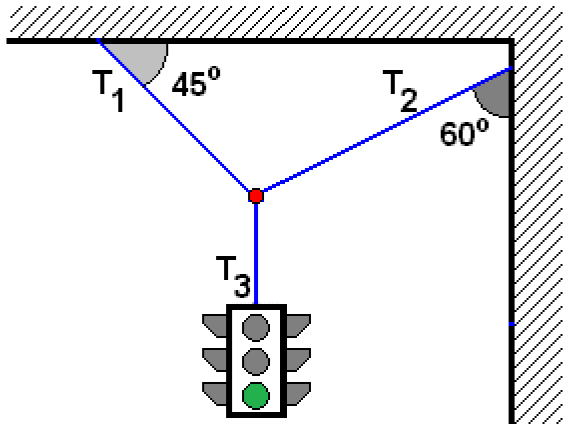
Physics I

Statics knots

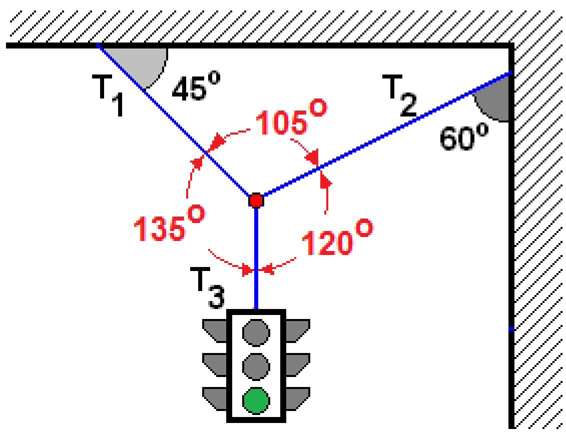
Equilibrium equations $\sum F = 0$ and $\sum \tau = 0$ Where $\tau = Fr\sin \angle_F^r$

Law of sines $\frac{F_1}{\sin\theta_1} = \frac{F_2}{\sin\theta_2} = \frac{F_3}{\sin\theta_3}$

Problem 1.- The mass of the traffic light in the figure is 40 kg, so the tension T_3 is 392N. Calculate the tensions T_1 and T_2.



Solution: We identify the angles between the vectors using geometry:

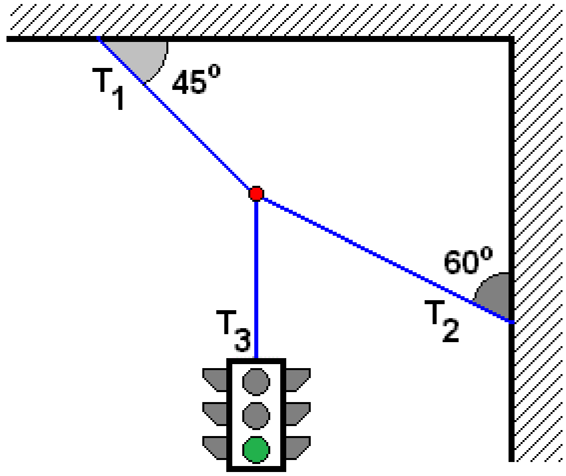


And now we can use the law of sines:

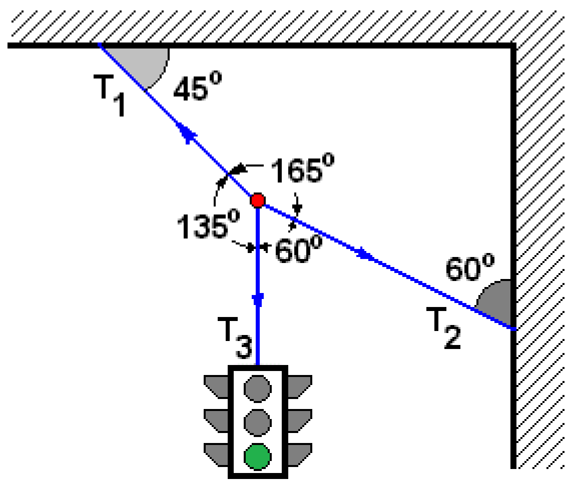
$$\frac{392}{\sin 105^\circ} = \frac{T_1}{\sin 120^\circ} \rightarrow T_1 = \mathbf{350\ N}$$

$$\frac{392}{\sin 105^\circ} = \frac{T_2}{\sin 135^\circ} \rightarrow T_2 = \mathbf{290\ N}$$

Problem 1a: The mass of the traffic light in the figure is 50 kg, so the tension T_3 is 490 N. Calculate the tensions T_1 and T_2.



Solution: We can determine the angles in the problem by using geometry:

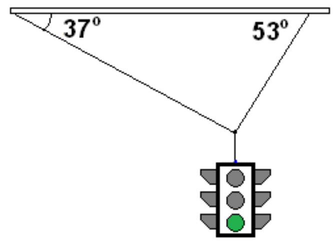


So, we know that the angle opposite to T_1 is 60 degrees, the one opposite to T_2 is 135 and the one opposite to T_3 is 165, then we can use the "law of sines":

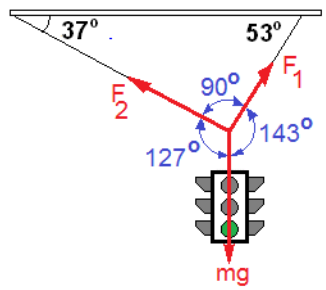
$$\frac{490}{\sin 165^\circ} = \frac{T_1}{\sin 60^\circ} \rightarrow T_1 = \frac{490 \sin 60^\circ}{\sin 165^\circ} = \mathbf{1{,}640\ N}$$

$$\frac{490}{\sin 165^\circ} = \frac{T_2}{\sin 135^\circ} \rightarrow T_2 = \frac{490 \sin 135^\circ}{\sin 165^\circ} = \mathbf{1{,}340\ N}$$

Problem 2.- Find the tension in the cables given that the mass of the traffic light is 50 kg.



Solution: We can solve the problem based on the law of sines:

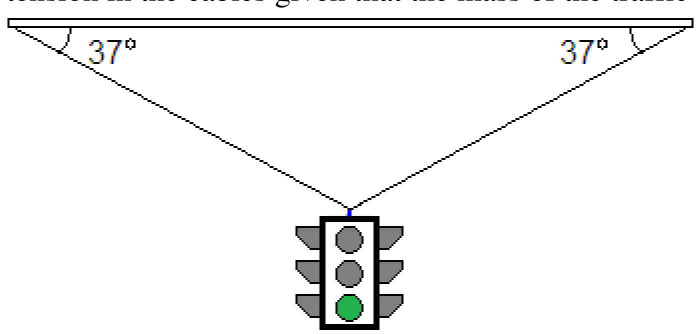


The relevant angles are shown in blue. And the equations are:

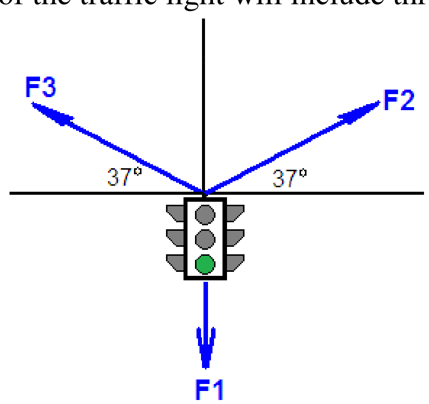
$$\frac{mg}{\sin 90^{\circ}} = \frac{F_1}{\sin 127^{\circ}} \rightarrow F_1 = mg \sin 127^{\circ} = \mathbf{391\ N}$$

$$\frac{mg}{\sin 90^{\circ}} = \frac{F_2}{\sin 143^{\circ}} \rightarrow F_1 = mg \sin 143^{\circ} = \mathbf{295\ N}$$

Problem 3.- Find the tension in the cables given that the mass of the traffic light is 88 kg.



Solution: A free body diagram of the traffic light will include three forces: F1, F2 and F3

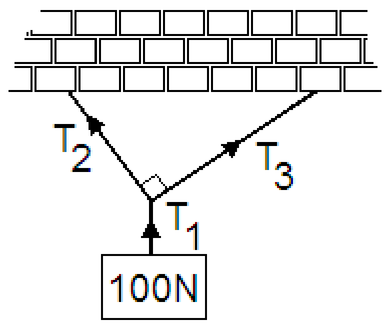


The two tensions have the same magnitude; you can realize this is the case by just considering the mirror symmetry of the problem.
To find how much this magnitude is, we can write the equilibrium equation for the y-axis:

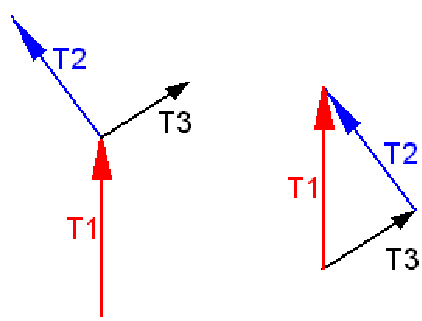
$$\sum \mathrm{F_y} = 0 \rightarrow \mathrm{F_2 Sin}(37^\circ) + \mathrm{F_3 Sin}(37^\circ) - Mg = 0$$

$$\rightarrow \mathrm{F_T} = \frac{\mathrm{Mg}}{2\mathrm{Sin}(37^\circ)} = \frac{88kg(9.8m/s^2)}{2(0.6)} = \textbf{720 N}$$

Problem 4.- Which tension is the smallest? Which one the largest? Defend your answer with a vector diagram or equations.

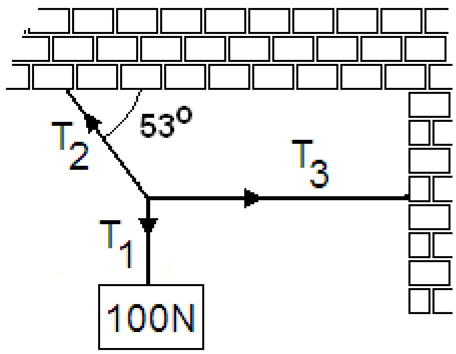


Solution: Focus on the knot where the three strings are tied together, since it is in equilibrium and has negligible mass, we know that $\vec{T}_1 = \vec{T}_2 + \vec{T}_3$. A graph of the three forces is shown in the figure:



By examining the triangle that the force vectors form we determine that T_1 is the largest force and T_3 is the smallest.

Problem 4a.- Find the tension on the strings.



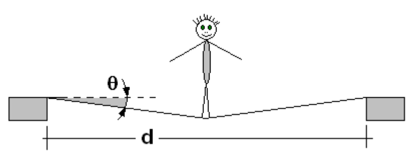
Solution: We know that F_{T1}=100 N, and we can use the "law of sines" to find the other forces:

$$\frac{100N}{\sin(180^\circ-\theta)}=\frac{F_{T2}}{\sin 90^\circ}\rightarrow F_{T2}=\frac{100N}{\sin\theta}$$
$$\frac{100N}{\sin(180^\circ-\theta)}=\frac{F_{T3}}{\sin(90^\circ+\theta)}\rightarrow F_{T3}=\frac{100N}{\sin\theta}\times\cos\theta$$

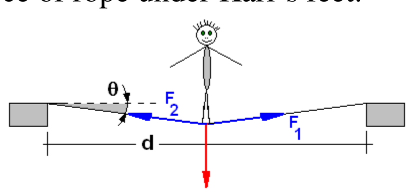
So, if $\theta=53^\circ$:

$F_{T2}=\dfrac{100N}{\sin 53^\circ}=$ **125 N** and $F_{T3}=\dfrac{100N}{\sin 53^\circ}\times\cos 53^\circ=$ **75 N**

Problem 5.- Karl Petit, a circus performer, walks across a "tight rope" strung horizontally between two supports separated by a distance **d=12 m**. The sag in the rope when he is at the midpoint is **θ=15º**. Calculate the tension in the rope at that point knowing that Karl Petit's mass is **m=55 kg**.



Solution: Consider the small piece of rope under Karl's feet.



There are three forces acting on that piece of rope:

$Weight = (0, -mg)$

$F_1 = (F\cos 15^\circ, F\sin 15^\circ)$ The tension on the right

$F_2 = (-F\cos 15^\circ, F\sin 15^\circ)$ The tension of the left

In the y-direction we get: $-mg + 2F\sin 15^\circ = 0 \to F = \dfrac{mg}{2\sin 15^\circ}$

For a mass of 55kg: $F = \dfrac{55(9.8)}{2\sin 15^\circ} =$ **1,041 N**