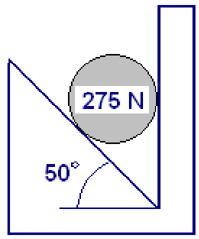
Physics I

Other Statics Problems

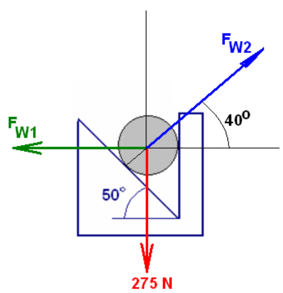
Equilibrium equations $\sum F = 0$ and $\sum \tau = 0$ Where $\tau = Fr\sin \angle_F^r$

Law of sines $\dfrac{F_1}{\sin\theta_1} = \dfrac{F_2}{\sin\theta_2} = \dfrac{F_3}{\sin\theta_3}$

Problem 1.- The sphere shown in the picture has a weight of 275 N and is resting against two smooth surfaces (neglect friction). Find the forces applied by the two surfaces.



Solution: The weight of the sphere is 275 N and it is a vector in the negative y-direction. The force due to the vertical wall will be in the negative x-direction. And the force due to the other wall will be in the first quadrant making an angle of 40º with respect to the positive x-axis as shown in the figure:



Writing down the vectors:

$F_{weight} = (0,-275N)$

$F_{W1} = (-F_{W1},0)$

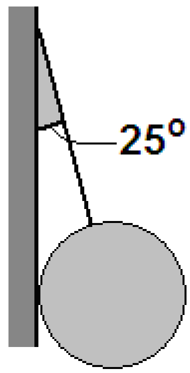
$F_{W2} = (F_{W2}\cos 40^\circ, F_{W2}\sin 40^\circ)$

Using the conditions of equilibrium:

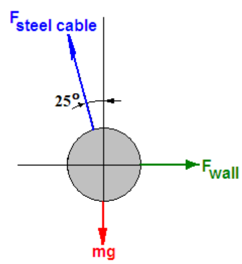
$$\sum F_y = 0 \rightarrow -275N + F_{W2}\sin 40^\circ = 0 \rightarrow F_{W2} = \frac{275N}{\sin 40^\circ} = \mathbf{427\ N}$$

$$\sum F_x = 0 \rightarrow -F_{W1} + F_{W2}\cos 40^\circ = 0 \rightarrow F_{W1} = F_{W2}\cos 40^\circ = 427N\cos 40^\circ = \mathbf{328\ N}$$

Problem 2.- The figure shows a 1500-kg sphere leaning against a frictionless wall and held in place by a steel cable. Find the force that the wall applies on the sphere.



Solution: The weight of the sphere is: mg=1500kg (9.8m/s^2) = 14,700 N.
The free body diagram of the sphere includes three forces: The weight pointing downward, the reaction from the wall, which is a horizontal force (because there is no friction) and the steel cable tension:



The components of the forces can be written as:

$$F_{weight} = (0, -14{,}700N)$$

$$F_{wall} = (F_{Wall}, 0)$$

$$F_{steel\ cable} = (-F_{SC}\sin 25^\circ, F_{SC}\cos 25^\circ)$$

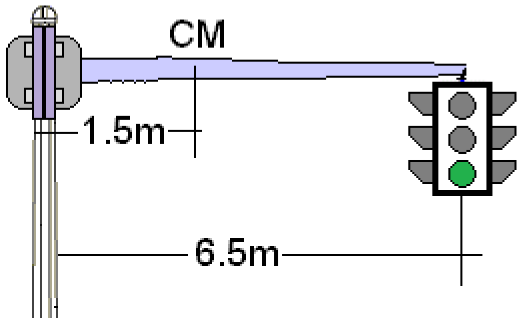
Since the sphere is in equilibrium, the sum of the forces is zero:

$$\sum F_y = 0 \rightarrow -14{,}700N + F_{SC}\cos 25^\circ = 0 \rightarrow F_{SC} = \frac{14{,}700N}{\cos 25^\circ}$$

And:

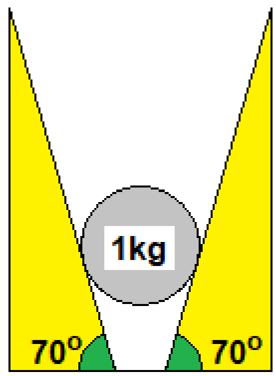
$$\sum F_x = 0 \rightarrow F_{Wall} - F_{SC}\sin(25^\circ) = 0 \rightarrow F_{Wall} = F_{SC}\sin 25^\circ = 14{,}700N\frac{\sin 25^\circ}{\cos 25^\circ} = \mathbf{6{,}850\ N}$$

Problem 3.- The mass of the traffic light is 45kg and the mass of the supporting pole is 125kg. Notice that the center of mass of the pole is not in the middle because it has a tapered shape. Calculate the torque needed to keep the whole thing in equilibrium.

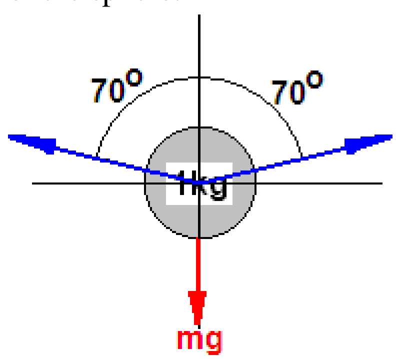


Solution: The traffic light produces a clockwise torque of 45kg(9.8m/s^2)(6.5m) = 2866 Nm. The supporting pole produces a clockwise torque of 125kg(9.8m/s^2)(1.5m) = 1837 Nm. So, we need **4,700 Nm** in the counterclockwise direction to keep the whole thing from rotating.

Problem 4.- Calculate the forces that the 1kg-sphere produces on the frictionless walls:



Solution: The free body diagram of the sphere:



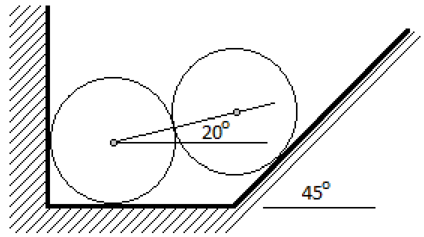
$$\vec{F}_{weight} = (0,-9.8N)$$

$$\vec{F}_1 = F(\sin 70^\circ, \cos 70^\circ)$$

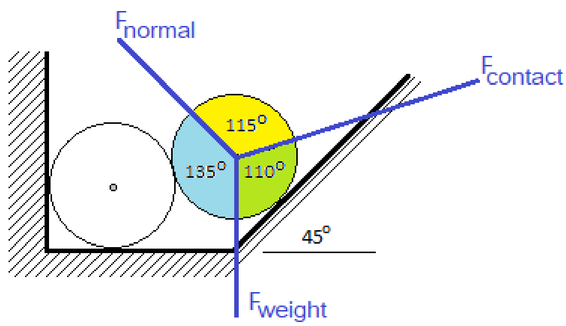
$$\vec{F}_2 = F(-\sin 70^\circ, \cos 70^\circ)$$

Using Newton's second law: $\sum F_y = -9.8N + 2F\cos 70^\circ \rightarrow F = \dfrac{9.8N}{2\cos 70^\circ} =$ **14.3N**

Problem 5.- Calculate the force between the sphere on the left and the vertical wall. Each sphere has 100kg of mass and all the surfaces are smooth (neglect friction).

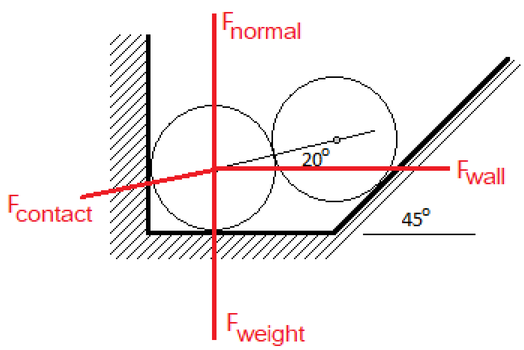


Solution: We first analyze the sphere on the right side. There are three forces acting on it, with the angles shown in the figure determined by geometry.



Using the law of sines, we can find the contact force with the other sphere.

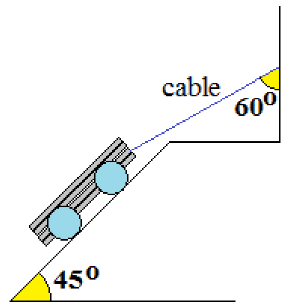
$$\frac{F_{contact}}{\sin 135^{\circ}} = \frac{F_{weight}}{\sin 115^{\circ}} \rightarrow F_{contact} = \frac{\sin 135^{\circ}}{\sin 115^{\circ}} F_{weight} = 764.6N$$



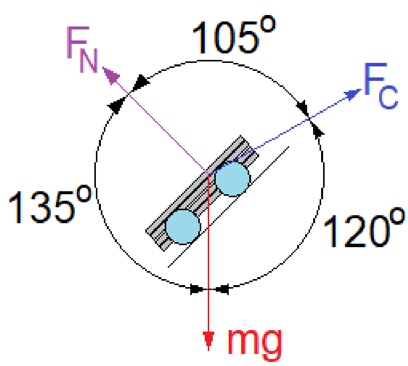
Observing the sphere on the left, we notice that the force from the wall must be equal to the horizontal component of the contact force between the spheres, so the force is

$$F_{wall} = 764.6N \cos 20^{\circ} = 718N$$

Problem 6.- Calculate the force in the cable if the weight of the cart is mg = 1250 N and there is no friction between the wheels and the inclined surface.



Solution: We draw a free body diagram of the cart. In doing this, we replace the normal forces acting on the wheels by a single force acting on the intersection of the weight and the tension in the cable.



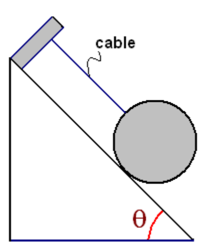
The angles between forces are determined from the geometry of the problem.

To find the tension force in the cable we can use the law of sines:

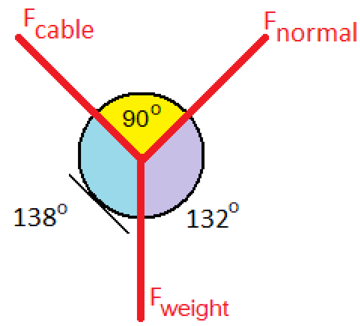
$$\frac{F_C}{\sin 135^\circ} = \frac{mg}{\sin 105^\circ} \rightarrow F_C = \frac{mg}{\sin 105^\circ}\sin 135^\circ = \mathbf{915\ N}$$

Problem 7.- The figure shows a 1800-kg sphere leaning against a smooth surface without friction and kept in place by a cable parallel to the inclined plane.

Draw the free body diagram of the sphere and calculate the force in the cable and the normal force if $\theta = 48^\circ$



Solution: A free body diagram with the angles between the forces is shown below.



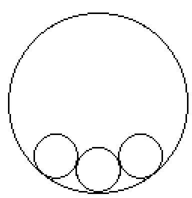
To find the forces, we use the law of sines:

$$\frac{1800kgf}{\sin 90^\circ} = \frac{F_{normal}}{\sin 138^\circ} = \frac{F_{cable}}{\sin 132^\circ}$$

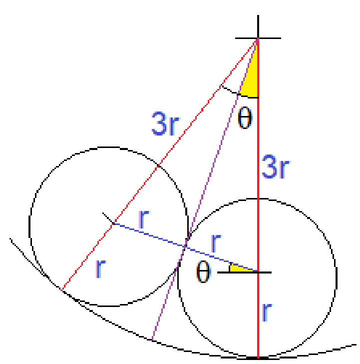
$$\rightarrow F_{normal} = 1{,}200kgf = 11{,}800N$$

$$\rightarrow F_{cable} = 1{,}340kgf = 13{,}100N$$

Problem 8.- The figure shows three cylinders with radii 30cm inside a pipe with 120cm radius. If each cylinder weighs 81N calculate the force between the central cylinder and the pipe. All surfaces are smooth (neglect friction).

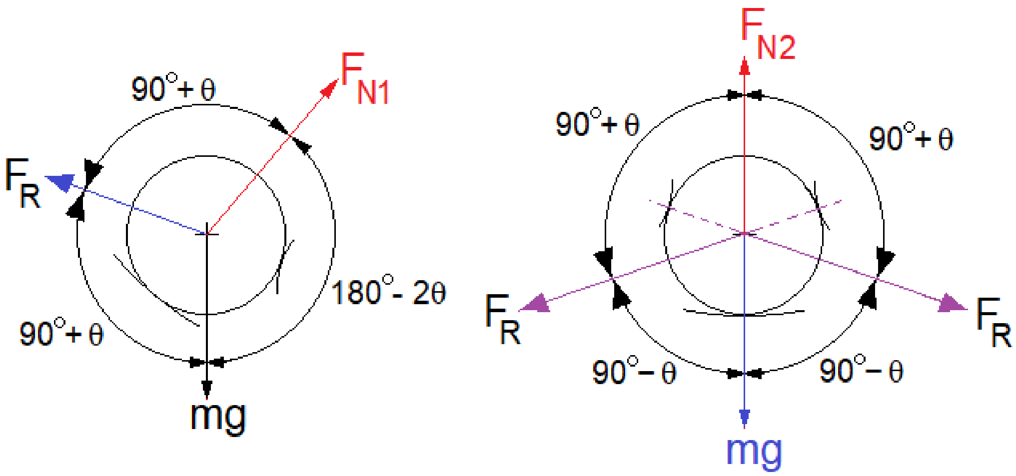


Solution: We can use trigonometry to find the relevant angles to the problem. We notice that the radius of the cylinders is ¼ of the pipe, so we can draw the following geometric construction:



Where $\theta = \sin^{-1}\left(\frac{r}{3r}\right) = 19.5^\circ$

Free body diagrams with the angles between the forces are shown below for the left and middle cylinders:



In the case of the left cylinder, since there are only three forces acting on it, we can use the law of sines to find the reaction force F_R, which is the contact force with the middle cylinder.

$$\frac{F_R}{\sin\left(180^\circ - 2\theta\right)} = \frac{mg}{\sin\left(90^\circ + \theta\right)} \rightarrow F_R = \frac{\sin\left(180^\circ - 2\theta\right)}{\sin\left(90^\circ + \theta\right)} mg = 2mg\sin\theta$$

For the middle cylinder we can focus on the vertical forces. To be in equilibrium we need:

$$F_N = mg + 2F_R \sin\theta$$

After replacing the calculated value of FR and some simplification we get:

$$F_N = mg(1 + 4\sin^2\theta) = 81N\left(1 + \frac{4}{9}\right) = \mathbf{117N}$$