Physics I

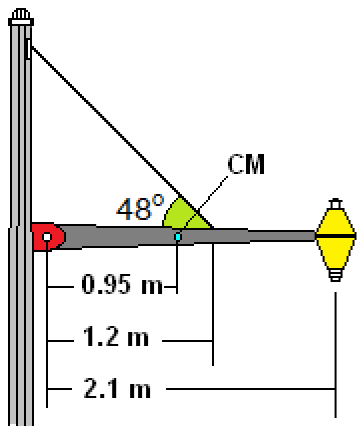
Statics with poles

First condition of equilibrium $\sum F_x = 0$ and $\sum F_y = 0$

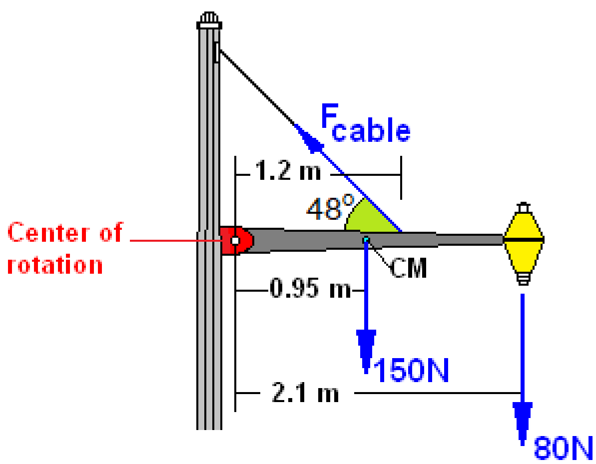
Second condition of equilibrium $\sum \tau = 0$ where $\tau = \pm F r \sin \angle_F^r$

Law of sines $\dfrac{F_1}{\sin\theta_1} = \dfrac{F_2}{\sin\theta_2} = \dfrac{F_3}{\sin\theta_3}$

Problem 1.- Find the tension in the cable. The weight of the lamp is 80N, the weight of the horizontal pole is 150N and its center of mass is 0.95 m from the left side (it is a tapered shape, so it is not in the middle).



Solution: Taking the hinge to be the center of rotation there are three forces that produce torque on the horizontal pole: The weight of the lamp, the weight of the pole and the tension in the cable:

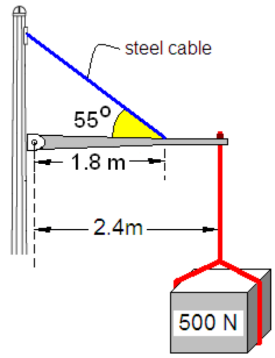


According to the second condition of equilibrium, the net torque of these three forces needs to be zero:

$$\sum \tau = F_{cable}(1.2m)\sin 48^\circ - 150N(0.95m) - 80N(2.1m) = 0$$

So the tension in the cable is: $F_{cable} = \dfrac{150N(0.95m) + 80N(2.1m)}{(1.2m)\sin 48^\circ}$ = **348N**

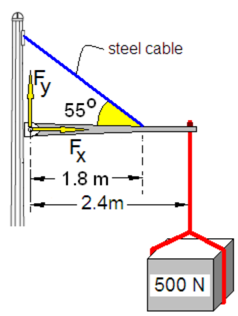
Problem 2.- Calculate the tension (force) in the cable. Neglect the weight of the pole.



Solution: We apply the second condition of equilibrium:

$$500\times 2.4 = F_T \times 1.8 \times \sin 55^\circ \rightarrow F_T = \frac{500\times 2.4}{1.8\times \sin 55^\circ} = \mathbf{814\ N}$$

Problem 2a.- Calculate the horizontal force (F_x) acting on the hinge. Neglect the weight of the pole.

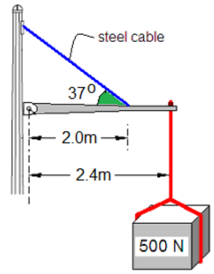


Solution: We first find the tension force in the cable. To do this we use the second condition of equilibrium ($\sum \tau = 0$), so:

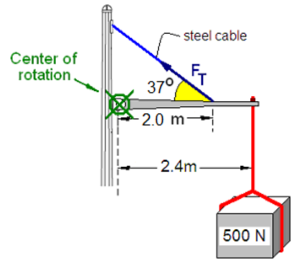
$$-500\times 2.4 + F_{cable} \times 1.8 \sin 55^\circ = 0 \rightarrow F_{cable} = 814N$$

And notice that the horizontal force F_x has to be equal to the horizontal component of the tension force, so: $F_x = 814 \cos 55^\circ =$ **467 N**

Problem 3.- Calculate the tension on the steel cable. Neglect the weight of the pole.



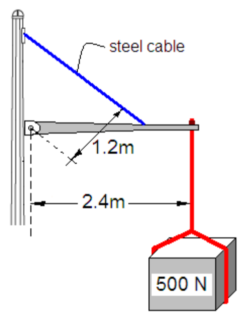
Solution: Choosing the hinge as the center of rotation:



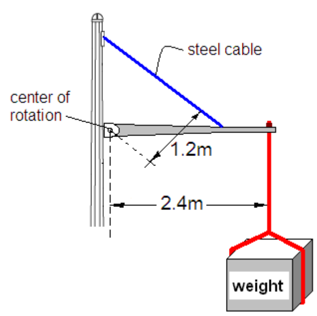
Using the second condition of equilibrium: $\sum \tau = 0$

$$-500 \times 2.4 + F_T \times 2.0 \times \sin 37^{\circ} = 0 \rightarrow F_T = \frac{500 \times 2.4}{2.0 \times \sin 37^{\circ}} = \textbf{1,000 N}$$

Problem 4.- Calculate the tension (force) on the cable. Neglect the weight of the pole.



Solution: If we choose the point of rotation to be the hinge, there are only two torque-producing forces: The weight and the tension in the cable.



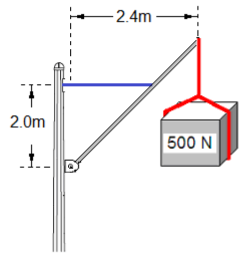
The pole is not rotating, so the sum of the torques applied is zero.

Torque due to the weight =500N × 2.4 m
Torque due to the cable = F × 1.2 m

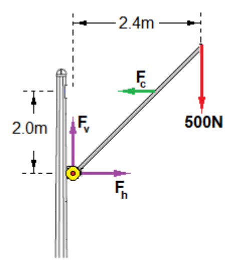
They must be equal, so:

F × 1.2 m = 500N × 2.4 m, then **F = 1,000 N**

Problem 5.- Calculate the tension force in the cable and the reaction at the hinge. The block has a weight of 500 N, and we can neglect the weight of the supporting pole.



Solution: We label the forces at the hinge F_h and F_v.



Then, if we choose the point of rotation to be the hinge, there are only two torque-producing forces: The weight and the tension in the cable. They need to add to zero:

$$F_C \times 2.0m - 500N \times 2.4m = 0 \rightarrow F_C = \frac{500 \times 2.4}{2.0} = 600N$$

Notice that we took the value of the arm lengths directly from the geometry of the problem.

The sum of the forces in the horizontal direction is zero, so

$$F_h - F_C = 0 \rightarrow F_h = 600N$$

And likewise for the vertical direction:

$$F_v - 500N = 0 \rightarrow F_v = 500N$$