Physics I

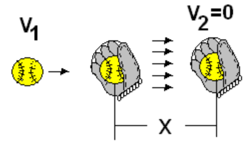
Force

Newton's second law of motion $\sum F_x = ma_x \qquad \sum F_y = ma_y$

Equations for constant acceleration

$$x = v_1 t + \frac{1}{2}at^2 \qquad v_2 = v_1 + at \qquad v_2^{\,2} = v_1^{\,2} + 2ax \qquad \bar{v} = \frac{v_1 + v_2}{2}$$

Problem 1.- A baseball (**mass = 0.141 kg**) traveling at **v_1=32.5 m/s** strikes the catcher's mitt, which brings the ball to rest by recoiling **x=0.155 m**. Calculate the average force acting on the glove.



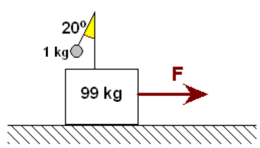
Solution: First let's find the acceleration:

$$v_2^{\,2} = v_1^{\,2} + 2ax \rightarrow a = \frac{v_2^{\,2} - v_1^{\,2}}{2x} = \frac{0^2 - 32.5^2}{2\times 0.155} = \text{-3407 m/s}^2$$

Now we use Newton's second law to find the force:

$$F = ma = 0.141\times 3407 = \mathbf{480\ N}$$

Problem 2.- A crate that has a mass of 99kg is pulled with a force F. A pendulum mounted on the crate has a mass of 1kg and hangs at an angle $\theta = 20^\circ$ off the vertical. Calculate the force F if the coefficient of friction between the crate and the floor is $\mu_k = 0.35$



A) 335 N B) 678 N C) 357 N D) 343N E) 700 N

Solution: The angle $\theta = 20^\circ$ off the vertical indicates that the acceleration of the 1kg pendulum (and the whole system) is:

$$a = g\tan\theta$$

Now, the normal force is equal to the total weight of the crate plus pendulum, so:

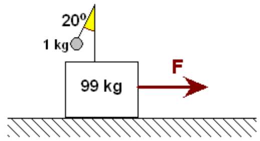
$$F_N = mg = 100\times 9.8 = 980N \rightarrow F_{friction} = \mu_K F_N = 0.35\times 980N = 343N$$

And Newton's second law is then:

$$F - F_{friction} = ma \rightarrow F = F_{friction} + ma = 343 + 100\times 9.8\tan 20^\circ = \mathbf{700\ N}$$

Answer: E

Problem 2a.- A crate that has a mass of 99kg is pulled with a force F. A pendulum mounted on the crate has a mass of 1kg and hangs at an angle $\theta = 20^{\circ}$ off the vertical. Calculate the force F if there is no friction between the crate and the floor.



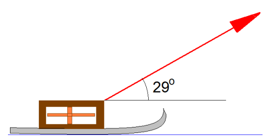
Solution: The angle $\theta = 20^{\circ}$ off the vertical indicates that the acceleration of the 1kg pendulum (and the whole system) is:

$$a = g \tan\theta$$

And Newton's second law is then:

$$F = ma \rightarrow F = 100 \times 9.8 \tan 20^{\circ} = \mathbf{360\ N}$$

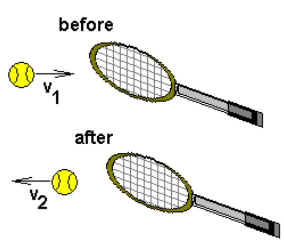
Problem 3.- A person pulls a 60 kg sled on an icy surface with a force of 80.0 N at an angle of 29º upward from the horizontal. Calculate the acceleration. Ignore friction in this problem.



Solution: Only the horizontal component of the external force produces acceleration so:

$$F \cos 29^{o} = ma \rightarrow a = \frac{F \cos 29^{o}}{m} = \frac{80 \cos 29^{o}}{60} = \mathbf{1.2\ m/s^2}$$

Problem 4.- In playing tennis a ball that had an initial velocity of v_1=26m/s horizontally is returned also horizontally with a speed of v_2=24m/s. Calculate the average force on the ball if its mass is 0.057 kg and the contact with the racket lasted 5 milliseconds.



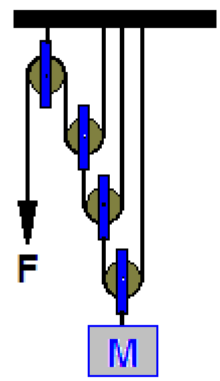
Solution: $F = \dfrac{\Delta p}{t} = \dfrac{50 \times 0.057}{0.005} = \mathbf{570\ N}$

Problem 5.- A 0.22 kg object follows the path given by $\vec{r} = (3\sin 2t,\ 4\cos 2t)$. Calculate the force acting on the object.

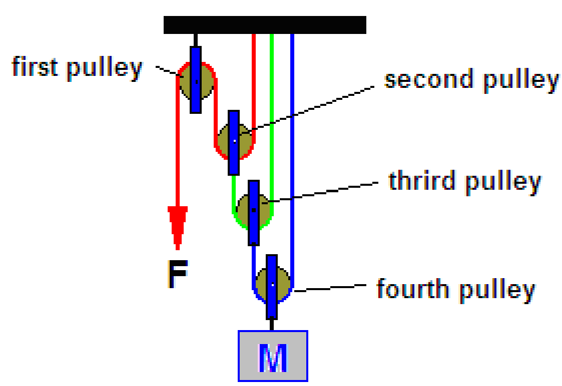
Solution: The acceleration is $a = \frac{d^2}{dt^2}\vec{r} = (-12\sin 2t, -16\cos 2t)$

The force is $F = ma = 0.22(-12\sin 2t, -16\cos 2t) =$ **(-2.64sin2t, -3.52cos2t)**

Problem 6.- What force (F) do you need to apply to lift the block of mass M shown in the figure?



Solution: The key to solve this problem is to look at each individual string considering that the tension is essentially the same anywhere. The following figure should help:



Notice that the tension in the red string is F. If you look at the second pulley neglecting its own weight, the tension in the green string must be twice the tension in the red string, because you have two red strings pulling up and one green string pulling down, so the tension on the green string is 2F.

In the same way, by looking at the third pulley we have two green strings pulling up that are compensated by one blue string pulling down, so the tension in the blue string is twice the tension in the green string, that is 4F.

Finally, two blue strings pull up the block of mass M, so 8F=Mg, giving **F=Mg/8**.

Problem 7.- A bullet of mass 2 g is shot horizontally into a sandbag, striking the sand with a velocity of 600 m/s. It penetrates 20 cm before stopping. What is the average stopping force acting on the bullet?

Solution: We know the initial velocity of the bullet (600m/s), its final velocity (zero) and the displacement (x=0.2m) so we can calculate the acceleration:

$$a = \frac{v_2^2 - v_1^2}{2x} = \frac{0 - (600 m/s)^2}{2(0.2 m)} = \text{-900,000 m/s}^2$$

To get the force we use Newton's second law: F=ma= 0.002kg(-900,000 m/s^2)= **- 1800 N**

Problem 8.- If the position of a 1.5 kg particle is described by the vector:

$$\vec{r} = (t^2, 5\cos t)$$

Find the net force acting on the particle as a function of time.

Solution Let us find the acceleration first:

$$\vec{r} = (t^2, 5\cos t)$$

$$\vec{v} = \frac{d\vec{r}}{dt} = (2t, -5\sin t)$$

$$\vec{a} = \frac{d\vec{v}}{dt} = (2, -5\cos t)$$

And the force is $\vec{F} = m\vec{a} = (3, -7.5\cos t)$

Problem 8a.- If the position of a 2.5 kg particle is described by the vector:

$$\vec{r} = (4t^4, 5\cos 2t)$$

Find the net force acting on the particle at time **t=1.57s**

Solution Let us find the acceleration first:

$$\vec{r} = (4t^4, 5\cos 2t)$$

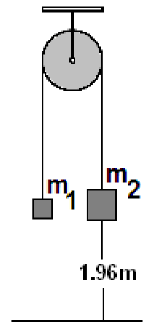
$$\vec{v} = \frac{d\vec{r}}{dt} = (16t^3, -10\sin 2t)$$

$$\vec{a} = \frac{d\vec{v}}{dt} = (48t^2, -20\cos 2t)$$

And the force is $\vec{F} = m\vec{a} = (120t^2, -50\cos 2t)$

Evaluated at t=1.57 we get $\vec{F} = m\vec{a} = (296, 50)$

Problem 9.- In the Atwood machine shown in the figure m_2=3 kg and m_1=2.5 kg and you can ignore the mass of the pulley and any friction. Find the speed of m_2 when it hits the ground if you release the masses with zero initial velocity.

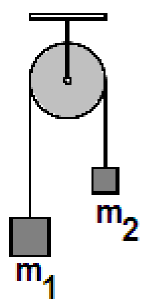


Solution: The acceleration is: $a = \dfrac{m_2 - m_1}{m_2 + m_1} g = \dfrac{3-2.5}{3+2.5} \times 9.8 = 0.89 m/s^2$

And the final velocity:

$v_2^{\;2} = v_1^{\;2} + 2ax = 0^2 + 2\times 0.89 \times 1.96 \rightarrow v_2 = \sqrt{2\times 0.89\times 1.96} =$ **1.9 m/s**

Problem 9a.- You want to study acceleration, but your rudimentary instruments only allow you to measure 1m/s^2 or less, so you build an Atwood machine to get less than that. What masses m_1 and m_2 would you choose to accomplish this?



Solution: The acceleration of the Atwood machine is given by:

$$a = \frac{M_2 - M_1}{M_2 + M_1} g$$

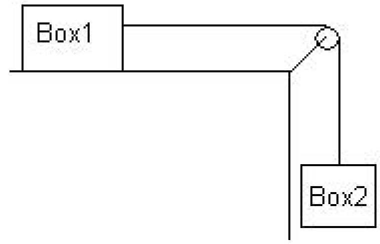
So if we want less than 1m/s^2 we should choose a fraction $\dfrac{M_2 - M_1}{M_2 + M_1}$ less than 1/9.8, for example

$M_2 = 1kg, M_1 = 0.9kg$ will give $\dfrac{M_2 - M_1}{M_2 + M_1} = \dfrac{0.1kg}{1.9kg} = 0.053$ which would work.

Problem 10.- Ignore friction in the following situation. Calculate how long it will take for box1 to slide 1.5 m if it starts from rest.

Mass of box1 = 1.1 kg

Mass of box2 = 2.2 kg



Solution: To find the acceleration we use Newton's second law, but we notice that only box2 is pulling the whole mass, so:

$$F = ma \rightarrow 2.2\times 9.8 = (2.2+1.1)a \rightarrow a = 6.53 \text{ m/s}^2$$

And now to find the time we use $x = v_1 t + \frac{1}{2}at^2$ to solve the problem:

$$1.5 = 0t + \frac{1}{2}6.53t^2 \rightarrow t = \sqrt{\frac{1.5\times 2}{6.53}} = \mathbf{0.67\ s}$$

Problem 11.- The brakes of an 800-kg car apply a force of -4,000N. Calculate the distance needed to stop the car if it is going at 35 miles per hour. [1 mile=1609 m]

Solution: The initial speed of the car in m/s is:

$$\mathrm{v_1} = 35\frac{\text{mile}}{\text{h}}\left(\frac{1\text{h}}{3600\text{s}}\right)\left(\frac{1609\text{m}}{1\text{mile}}\right) = 15.64\text{m/s}$$

The acceleration can be calculated using Newton's second law: $\mathrm{a} = \frac{\mathrm{F}}{\mathrm{m}} = \frac{-4000\mathrm{N}}{800\mathrm{kg}} = -5\mathrm{m/s^2}$

We find the distance using the kinematic equation: $\mathrm{v_2}^2 = \mathrm{v_1}^2 + 2\mathrm{ax}$

$$0^2 = 15.64^2 + 2(-5)x \rightarrow x = \frac{15.64^2}{2(5)} = \mathbf{24.5\ m}$$

Problem 12.- In an experiment with a force table you determine three forces:

F_1= 5N direction = 30°
F_2= 8N direction = 120°
F_3= 10N direction = 150°

Calculate the sum of these three vectors. Give your answer in magnitude and angle.

Solution: Let's represent the vectors as components:

$$A = (5\cos 30^\circ, 5\sin 30^\circ) = (4.33, 2.5)$$

$$B = (8\cos 120^\circ, 8\sin 120^\circ) = (-4, 6.93)$$

$$C = (10\cos 150^\circ, 10\sin 150^\circ))) = (-8.66, 5)$$

To get $\vec{A}+\vec{B}+\vec{C}$ we add the components of A, B and C:

$\vec{A}+\vec{B}+\vec{C} = (-8.33,\ 14.43)$

The magnitude of this vector is: $\sqrt{8.33^2+14.43^2} =$ **16.7 m**

And the angle:

$\tan^{-1}\left(\frac{14.43}{-8.33}\right) = -60^{\circ}$, but we add 180º because it is in the second quadrant: **120º**

Problem 13.- An aircraft carrier has a very short runway only 85m long. How much force would you need to apply to a 12,000 kg airplane for it to reach its final take off speed of 55m/s starting from rest?

Solution: $55^2 = 2a(85) \rightarrow a = 17.8$ and $F = ma = 12000\times 17.8 =$ **210,000 N**