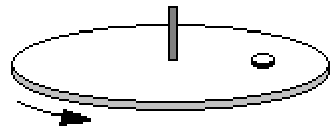
Physics I

Friction

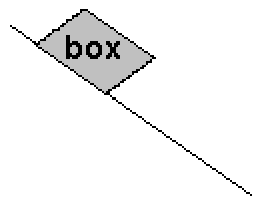
Problem 1.- A coin is located 0.2 m away from the center of a horizontal turntable which accelerates slowly. Calculate at what angular velocity (ω) the coin will start sliding if the coefficient of static friction is μ=0.5



Solution: The coin will start sliding when the friction force is not enough to produce the required centripetal force. At that point: $F_{friction} = \mu F_{Normal} = \mu mg = m\omega^2 R$, so to find ω:

$$\mu mg = m\omega^2 R \rightarrow= \omega = \sqrt{\frac{\mu g}{R}} = \sqrt{\frac{0.5\times 9.8}{0.2}} = \mathbf{4.9\ rad/s}$$

Problem 2.- A block begins to slide down a ramp after being elevated to an angle of 35 degrees. What is the coefficient of static friction?



Solution: At the point of sliding the component of the weight parallel to the slope (mgsinθ) matches the normal force times the coefficient of static friction (μmgcosθ), so:

$$mg\sin\theta = \mu mg\cos\theta \rightarrow \mu = \tan\theta = \tan 35^\circ = \mathbf{0.7}$$

Problem 3.- What is the maximum acceleration a car can undergo on a level road if the coefficient of static friction between the tires and the pavement is 0.65?

Solution:
Since the road is leveled the normal force is equal to the weight of the car and the maximum friction will be:

$$F_{FrictionMAX} = \mu_S F_N = 0.65mg$$

if we divide by the mass we get the maximum acceleration:

$$a_{MAX} = \frac{0.65mg}{m} = 0.65g = \mathbf{6.4 m/s^2}$$

Problem 3a.- What is the minimum distance to stop a car with an initial velocity of 15 m/s on a level road if the coefficient of static friction between the tires and the pavement is 0.65?

Solution: Since the road is leveled, the normal force is equal to the weight of the car and the maximum friction will be:

$$F_{FrictionMAX} = \mu_S F_N = 0.65mg$$

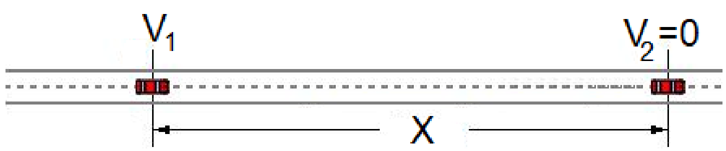
if we divide by the mass, we get the maximum acceleration:

$$a_{MAX} = \frac{0.65mg}{m} = 0.65g = 6.4\text{m/s}^2$$

To find the distance we use the equation $v_2^{\ 2} = v_1^{\ 2} + 2ax$

$$0^2 = 15^2 + 2(-6.4)x \rightarrow x = \frac{15^2}{2\times 6.4} = \mathbf{17.6\ m}$$

Problem 4.- Rubber on ice has a very low coefficient of kinetic friction $\mu_K = 0.13$
Based on this, calculate the distance needed to stop a car that is going at 20 miles per hour on a level icy road. Assume that the driver slam on the brakes, locking the wheels.
[1 mile=1609 m]



Solution: The speed of the car in m/s is:

$$v = 20\frac{mile}{h}\left(\frac{1h}{3600s}\right)\left(\frac{1609m}{1mile}\right) = 8.94m/s$$

The friction force is given by:

$$F_{fr} = \mu_S F_{Normal} = \mu_S mg$$

To stop the car the kinetic energy is totally lost, so the work done by the friction force will be equal to the loss:

$$F_{fr}d = \mu_S mgd = \frac{1}{2}mv^2 \rightarrow d = \frac{\frac{1}{2}mv^2}{\mu_S mg} = \frac{v^2}{2\mu_S g}$$

For a coefficient of 0.13 we get: $d = \dfrac{(8.94m/s)^2}{2(0.13)(9.8m/s^2)} =$ **31.4 m**

Problem 5.- A hockey puck is given an initial speed of 2.5m/s and it slides 15m on the ice before it stops. Calculate the coefficient of kinetic friction.

Solution: The puck slows down due to the friction force: $F_{friction} = \mu_K F_{Normal} = ma$, but the normal force is equal to the weight in this case (level surface, no other vertical forces).

$$\mu_K mg = ma \rightarrow \mu_K = \frac{a}{g}$$

We know the initial velocity of the puck (2.5m/s) and the final velocity (zero) and the displacement (x=15m) so we can calculate the acceleration:

$$a = \frac{v_2^2 - v_1^2}{2x} = \frac{0-(2.5m/s)^2}{2(15m)} = \text{-0.208 m/s}^2$$

So, the coefficient of kinetic friction is: $\mu_K = \frac{a}{g} = \frac{0.208}{9.8} =$ **0.021**

Problem 6.- A 15 kg box is sitting on a rough, level surface. A horizontal force of 95 N is needed to start moving the box. Once the box starts moving the 95N force is maintained and the box accelerates at 1.5m/s^2

a) Find the coefficient of static friction.
b) Find the coefficient of kinetic friction.

Solution:

a) At the threshold point the friction force is 95N and the normal force (equal to the weight in this case) is 15×9.8=147 N, so the equation is

$$F_{friction} = \mu_S F_{Normal} \rightarrow \mu_S = \frac{F_{friction}}{F_{Normal}} = \frac{95}{147} = \mathbf{0.65}$$

b) Once the box is in motion, we can calculate the friction force using Newton's second law as follows

$$95 - F_{friction} = ma \rightarrow F_{friction} = 95 - \text{ma} = 95 - 15\times 1.5 = 72.5N$$

And now, we can calculate the coefficient of friction:

$$F_{friction} = \mu_k F_{Normal} \rightarrow \mu_k = \frac{F_{friction}}{F_{Normal}} = \frac{72.5}{147} = \mathbf{0.49}$$