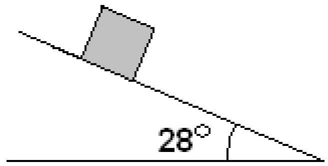
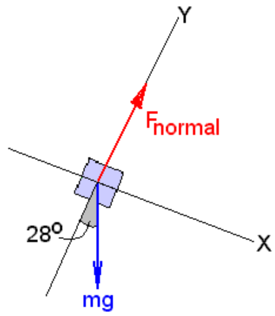
Physics I

Inclines

Problem 1.- Draw a free body diagram of the box shown in the figure and calculate its acceleration if it is sliding down the slope without friction.



Solution: Free body diagram:



In the rotated set of axes, the forces can be written as:

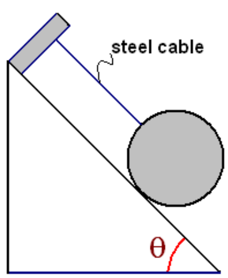
$\vec{F}_{weight} = (mg\sin 28^\circ, -mg\cos 28^\circ)$

$\vec{F}_{normal} = (0, F_{normal})$

In the x-axis using Newton's second law:

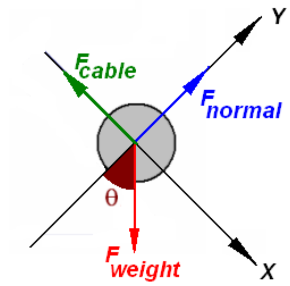
$\sum F_x = ma_x = mg\sin 28^\circ \rightarrow a_x = g\sin 28^\circ =$ **4.6 m/s²**

Problem 2.- The figure shows a 1500-kg sphere leaning against a frictionless surface and held in place by a steel cable which is parallel to the surface. Find the tension on the cable if the angle $\theta = 48^\circ$



Solution: The weight of the sphere is: mg = 1500kg (9.8m/s^2) = 14,700 N.

The free body diagram of the sphere (shown below) includes three forces: its weight, the normal force from the surface and the tension in the cable. They cancel to be in equilibrium.



We choose a rotated set of axes XY, where the components of the forces can be written as:

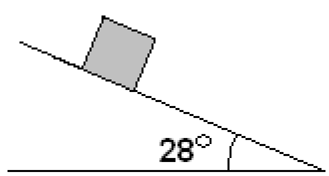
$$F_{weight} = (14{,}700\text{N}\sin\theta, -14{,}700\text{N}\cos\theta)$$
$$F_{normal} = (0, F_{normal})$$
$$F_{cable} = (-F_{cable}, 0)$$

Since the sphere is in equilibrium the sum of the forces is zero:

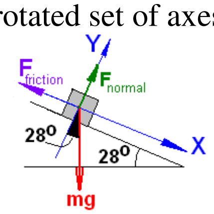
$$\sum F_x = 0 \rightarrow 14{,}700N\sin\theta - F_{cable} = 0 \rightarrow F_{cable} = 14{,}700N\sin 48^\circ = \mathbf{10{,}900\ N}$$

Problem 3.- The box in the picture has a mass of 15kg and it is in equilibrium (its acceleration is zero).

i) Draw the free body diagram of the object, showing all the forces and angles.
ii) Calculate the friction force.



Solution: There are three forces acting on the box: Its weight, the normal force, and the friction force. Since the object is in equilibrium the sum of the forces is zero.
The free body diagram is shown with a rotated set of axes:



Notice that in this rotated set of axes the three vectors can be written as follows:

$$F_{weight} = (mg\sin\theta, -mg\cos\theta)$$
$$F_{normal} = (0, F_N)$$
$$F_{friction} = (-F_{friction}, 0)$$

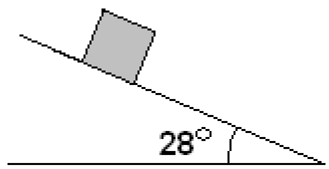
The sum of these vectors is zero, so in the Y direction:

$$-mg\cos\theta + F_N = 0 \rightarrow F_N = mg\cos\theta$$

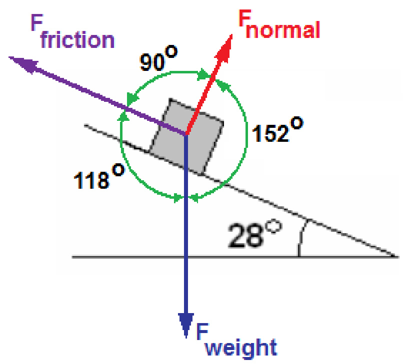
In the X direction:

$$mg\sin\theta - F_{friction} = 0 \rightarrow F_{friction} = mg\sin\theta = 15 \times 9.8 \sin 28^\circ = \mathbf{69\ N}$$

Problem 3a.- The box in the picture has a mass of 15kg and is sliding down with zero acceleration. Draw the free body diagram of the object, showing all the forces, and calculate the friction force.



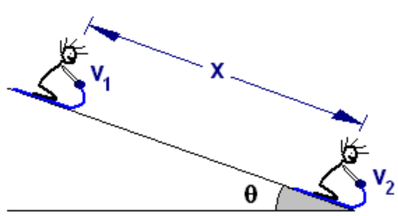
Solution: We first draw a free body diagram with the three forces acting on the box indicating the angles between them.



To solve the problem, we can use the law of sines because the forces add to zero:

$$\frac{F_{friction}}{\sin 152^\circ} = \frac{F_{weight}}{\sin 90^\circ} \rightarrow F_{friction} = \sin 152^\circ \frac{15 \times 9.8}{\sin 90^\circ} = \mathbf{69\ N}$$

Problem 4.- A sled starts down a slope with an initial velocity $\mathbf{v_1 = 4m/s}$. Calculate its final velocity after sliding $\mathbf{x = 12m}$ if the angle of the incline is $\boldsymbol{\theta = 22^o}$ and the coefficient of kinetic friction is $\boldsymbol{\mu_k = 0.2}$



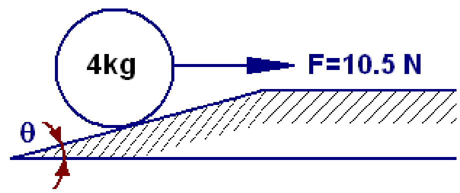
Solution: First, we calculate the acceleration

$$a = g\sin\theta - \mu g\cos\theta = 9.8\sin 22^\circ - 0.2\times 9.8\cos 22^\circ = 1.85 m/s^2$$

and now we calculate the final velocity:

$$v_2^{\ 2} = v_1^{\ 2} + 2ax \rightarrow v_2 = \sqrt{v_1^{\ 2} + 2ax} = \sqrt{4^2 + 2\times 1.85\times 12} = \mathbf{7.8\ m/s}$$

Problem 5.- Calculate the angle θ, knowing that the horizontal force necessary to keep the sphere from moving is 10.5 N.
Consider the friction between the sphere and the inclined surface to be zero.

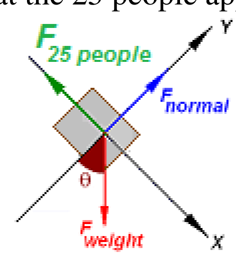


Solution: It can be shown that the tension in the string is the weight of the sphere times the tangent of the angle θ, so knowing that the horizontal force is 10.5 N we get:

$$10.5 = mg\tan\theta \rightarrow \theta = \tan^{-1}\left(\frac{10.5}{4\times 9.8}\right) = \mathbf{15.0^o}$$

Problem 6.- In building the pyramids of Egypt a theory proposes that 25 people would pull a 2,500 kg block up an incline at a 28° angle. Neglecting friction estimate the force applied by each person.

Solution: The weight of the block is: mg=2,500kg (9.8m/s^2)
The free body diagram of the block (shown below) includes three forces: its weight, the normal force from the surface and the force that the 25 people applied. They cancel to be in equilibrium.



We choose a rotated set of axes, where the components of the forces can be written as:

$$F_{weight} = (mg\sin\theta, -mg\cos\theta)$$

$$F_{normal} = (0, F_{normal})$$

$$F_{25people} = (-F_{25people}, 0)$$

Since the block is in equilibrium the sum of the forces is zero:

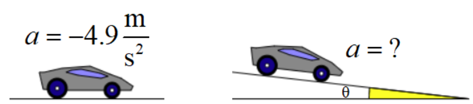
$$\sum F_x = 0 \rightarrow mg\sin\theta - F_{25people} = 0 \rightarrow F_{25people} = mg\sin\theta$$

With the values of the problem:

$$F_{25people} = 2500 \times 9.8 \times \sin 28^{\circ} = 11{,}500 \text{ N}$$

So, each person will pull: 24,500/25 **= 460 N**

Problem 7.- On a level road a car can decelerate at $a = -4.9\frac{\text{m}}{\text{s}^2}$ without skidding. With that information calculate the maximum possible deceleration if the road is inclined $\theta = 6.4^{\circ}$ downhill. Assume the value of μ_s is the same.

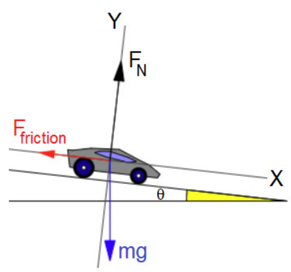


Solution: We use the information that the deceleration on level ground is $a = -4.9\frac{\text{m}}{\text{s}^2}$. This tells us the value of μ_s :

$$\mu_s = \frac{F_{friction}}{F_N} = \frac{ma}{mg} = \frac{a}{g} = \frac{4.9}{9.8} = 0.5$$

Notice that on a level road the normal force is equal to the weight and the friction force is the one calculated using Newton's second law.

Now that we know the coefficient of friction, we can solve the problem with the incline. To do this, consider the three forces acting on the car: Its weight, the normal force, and the friction force.



These three forces can be written in terms of their components as follows:

$$\vec{F}_N = (0, F_N)$$

$$\text{Weight} = (mg\sin\theta, -mg\cos\theta)$$

$$\vec{F}_{friction} = (-F_{friction}, 0)$$

Now, we use Newton's second law to solve the problem. Consider the y-direction first, where there is no acceleration, so the sum of the forces must be zero, then:

$F_N - mg\cos\theta = 0$, so: $F_N = mg\cos\theta$ … equation (1)

In the x-direction we do have acceleration, so Newton's second law should be written:

$mg\sin\theta - F_{friction} = ma_x$

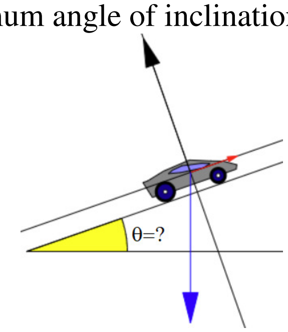
And knowing that $F_{friction} = \mu F_N$ we get:

$mg\sin\theta - \mu F_N = ma_x$ … equation (2)

Replacing equation (1) in equation (2): $mg\sin\theta - \mu mg\cos\theta = ma_x$
After simplifying: $a_x = g\sin\theta - \mu g\cos\theta$

If the angle is $\theta = 6.4^\circ$ we get: $a_x = 9.8\sin 6.4^\circ - 0.5\times 9.8\cos 6.4^\circ =$ **-3.8 m/s^2**

Problem 8.- Take the coefficient of static friction between rubber and wet asphalt to be **μ_s= 0.35,** with these conditions find the maximum angle of inclination that a car can climb.



Solution: According to the results of equilibrium, the friction force has to match the component of the weight in the x-direction and that means:

For $\mu = 0.35$ $\theta = \tan^{-1} 0.35 =$ **19.3º**