Physics I

More dynamics problems

Newton's second law $\sum F_x = ma_x \qquad \sum F_y = ma_y$

Problem 1.- A coin is thrown sliding upwards on an inclined plane 30º and it decelerates at a rate of 6 m/s^2. What will be its acceleration when it slides down? Approximate g = 10m/s^2

A) 2 m/s^2 B) 3 m/s^2 C) 4 m/s^2 D) 5 m/s^2 E) 6 m/s^2

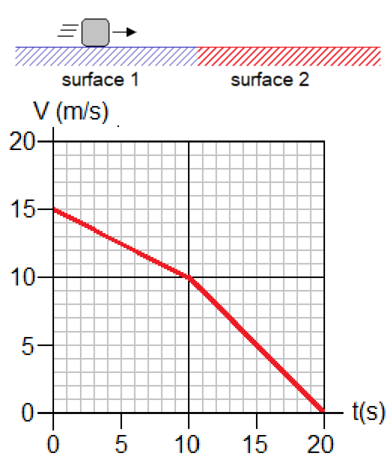
Solution: without friction, the deceleration should be 10 sin 30º = 5 m/s^2, bu friction adds 1m/s^2 to give the indicated 6 m/s^2.

When sliding down, the friction force will be opposed to the velocity, so the new acceleration will be:

5 m/s^2-1 m/s^2 = 4 m/s^2

Answer: **D**

Problem 2.- The graph shows the velocity of a block as a function of time. Determine the friction coefficient of the block with surface 2. Approximate g = 10m/s^2

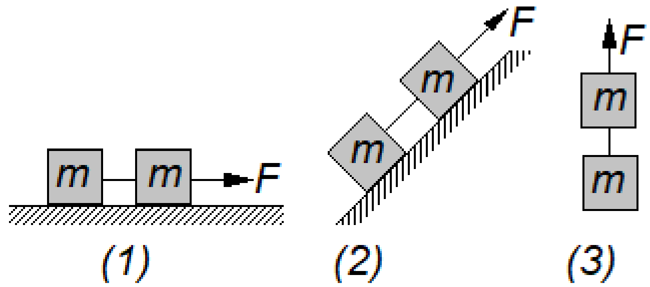


A) 0.01 B) 0.02 C) 0.03 D) 0.05 E) 0.10

Solution: We notice that in the second surface the block decelerates at a rate of 1m/s^2. This indicates that the friction force is m(1m/s^2) and since the normal force is mg, the friction coefficient is μ = (1m/s^2)/ (10m/s^2) = 0.1

Answer: **E**

Problem 3.- In the following cases there is no friction, and the force F is the same. Analyze in which case the tension of the string between the masses is maximum.



Solution: We analyze case by case:

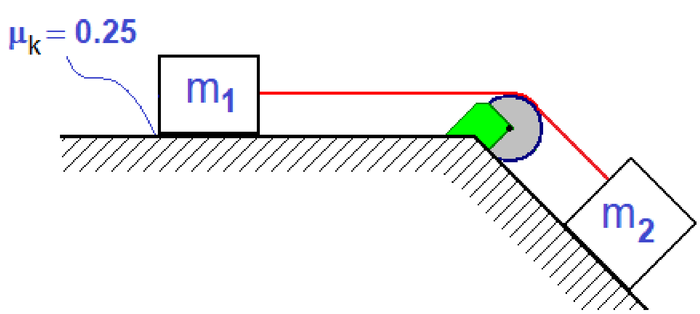
(1) The weight is cancelled by the normal force in each mass, but they are accelerated towards the right with acceleration F/(2m). Hence, the tension in the string that joins them is F/2.

(2) The normal force on each mass cancels the component of the weight perpendicular to the inclined plane, but the component parallel to the plane minus the force F acts on both masses producing an acceleration of $(2mg \sin \theta - F)/(2m)$. Again, the force in the string that joins the masses is F/2.

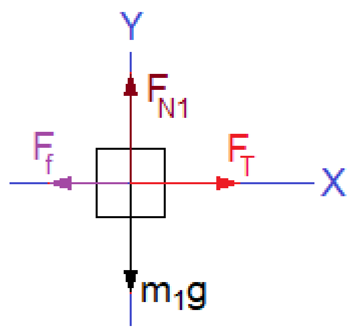
(3) The masses accelerate downwards with acceleration (2mg-F)/(2m) and that means that once again, the tension in the string that joins the two masses is F/2.

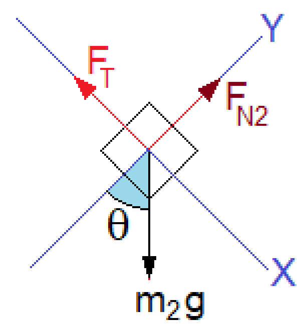
In conclusion, the force is F/2 in all three cases. Can you think of a better way of noticing this?

Problem 4.- In the following problem, m_1 = 2.0kg, m_2 = 5.0kg, μ_k = 0.25, and the angle of the incline is 45°. Calculate the tension in the string when mass m_1 is sliding to the right. Notice that there is only friction between m_1 and the horizontal surface.



Solution.- We start by drawing free body diagrams of the two masses:





Then, we apply Newton's second law to the two masses.

For m_1 we notice that there is no acceleration in the Y-direction, so the sum of the forces must be zero:

$$\sum F_y = 0 \rightarrow F_{N1} - m_1 g = 0 \rightarrow F_{N1} = m_1 g$$

However, m_1 is accelerated to the right with acceleration *a*, so:

$$\sum F_x = m_1 a \rightarrow F_T - F_f = m_1 a$$

The friction force can be calculated with $F_f = \mu_k F_N = \mu_k m_1 g$, so the equation becomes

$$F_T - \mu_k m_1 g = m_1 a \qquad (1)$$

For m_2 the weight can be decomposed into two forces, $m_2 g \sin\theta$ in the X-direction and $-m_2 g \cos\theta$ in the Y-direction. The Y-direction component is cancelled by the normal force, so we can write:

$$F_{N2} = m_s g \cos\theta$$

In the X-direction m_2 is accelerated with the same acceleration as m_1, so:

$$m_2 g \sin\theta - F_T = m_2 a \qquad (2)$$

We can combine equations (1) and (2) to get the acceleration. To do this, we sum the two equations side by side:

$$m_2 g \sin\theta - \mu_k m_1 g = (m_1 + m_2)a \rightarrow a = \frac{m_2 g \sin\theta - \mu_k m_1 g}{m_1 + m_2}$$

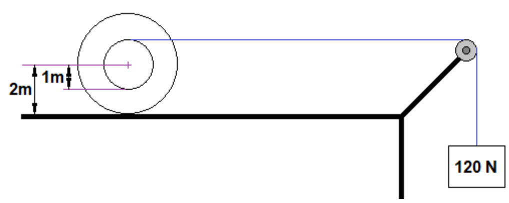
Plugging this result in equation (1) we get the tension in the string.

$$F_T = \frac{m_2 m_1}{m_1 + m_2}\left(\sin\theta + \mu_k\right) g$$

With the values of the problem

$$F_T = \frac{5\times 2}{5+2}\left(\sin 45^\circ + 0.25\right)9.8 = \mathbf{13.4N}$$

Problem 5.- When the system shown in the figure is let go from rest, the 120N weight goes down and the 200N solid cylinder rolls without slipping. Find the velocities of the weight and cylinder after the weight has dropped 3.88 meters.



Solution: Consider that the cylinder rolls a full turn. In that roll, its center will have moved a distance of 4π meters to the right and the string will have unraveled 2π meters, which means the weight will have dropped 6π meters. In other words, the relation between the displacements of the cylinder and the weight is

$$x_C = \frac{4}{6} x_W$$

Similarly, the relation between their center of mass speeds is

$$v_C = \frac{4}{6} v_W$$

Finally, since the cylinder rolls without slipping:

$$\omega_C = \frac{v_C}{2\text{m}} = \frac{4}{6}\frac{v_W}{2\text{m}}$$

To solve the problem, we consider that the potential energy lost by the weight is converted to kinetic energy of the weight and the cylinder (linear and rotational):

$$m_W g h = \frac{1}{2} m_W v_W{}^2 + \frac{1}{2} m_C v_C{}^2 + \frac{1}{2} I_C \omega_C{}^2$$

We can put all the velocities in terms of the velocity of the weight:

$$m_W gh = \frac{1}{2} m_W v_W{}^2 + \frac{1}{2} m_C \left(\frac{4}{6} v_W \right)^2 + \frac{1}{2} I_C \left(\frac{4}{6} \frac{v_W}{2\text{m}} \right)^2 \qquad (1)$$

In addition, we know that the moment of inertia of the cylinder is $I_C = m_C \frac{(2\text{m})^2}{2}$ and the mass of the cylinder is $m_C = \frac{200}{120} m_W$. Replacing these equations in (1) we get:

$$m_W gh = \frac{1}{2} m_W v_W{}^2 + \frac{1}{2} \left(\frac{200}{120} m_W \right) \left(\frac{4}{6} v_W \right)^2 + \frac{1}{2} \left(\frac{200}{120} m_W \frac{(2\text{m})^2}{2} \right) \left(\frac{4}{6} \frac{v_W}{2\text{m}} \right)^2$$

Solving for v_W, we get:

$$v_W = \sqrt{\frac{18}{19} gh} = \sqrt{\frac{18}{19} 9.8(3.88)} = \mathbf{6.00\ m/s}$$

And the velocity of the cylinder is:

$$v_C = \frac{4}{6} v_W = \mathbf{4.00\ m/s}$$