Physics I

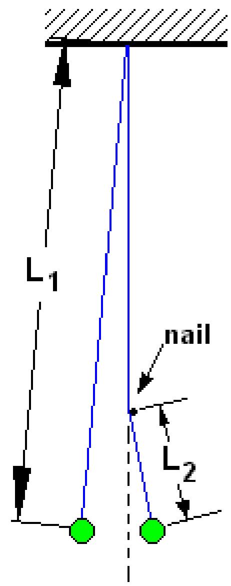
Pendulum

Period of a simple pendulum $T = 2\pi\sqrt{\frac{L}{g}}$

Period of a physical pendulum: $T = 2\pi\sqrt{\frac{I}{mg\ell}}$

Where I is the moment of inertia with respect to the point of rotation and ℓ is the distance from the point of rotation to the center of mass.

Problem 1.- A pendulum string has a total length L_1=1.2 m, but there is a nail which restricts the swing on the right side to only L_2=0.3m. Calculate the period of small oscillations of the pendulum.

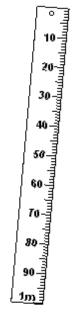


Solution: This pendulum will have a period equal to half the period corresponding to the length L_1=1.2 m and half to L_2=0.3m.

$$T = \frac{T_1}{2} + \frac{T_2}{2} = \pi\sqrt{\frac{L_1}{g}} + \pi\sqrt{\frac{L_2}{g}} = = \pi\sqrt{\frac{1.2}{9.8}} + \pi\sqrt{\frac{0.3}{9.8}} = \mathbf{1.65\ s}$$

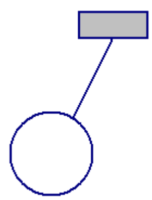
Problem 2.- Find the period of oscillation of a meter stick rotated about one end. Approximate it as if it were a thin rod.

[Moment of inertia of a rod rotated about one end $\mathrm{I} = \frac{\mathrm{ML}^2}{3}$]



Solution: $\mathrm{T} = 2\pi\sqrt{\frac{\mathrm{I}}{\mathrm{mg}\ell}} = 2\pi\sqrt{\frac{\frac{\mathrm{ML}^2}{3}}{\mathrm{Mg}\frac{L}{2}}} = 2\pi\sqrt{\frac{2\mathrm{L}}{3g}} = 2\pi\sqrt{\frac{2\times 1}{3\times 9.8}} = \mathbf{1.63s}$

Problem 3.- Calculate the period of small oscillations of a physical pendulum made of a hoop of radius 0.1m and mass M connected to a rod of negligible mass and length 0.2m.
[Moment of inertia of a hoop about its center=MR^2]



Solution: Period of a physical pendulum: $T = 2\pi\sqrt{\frac{I}{mg\ell}}$

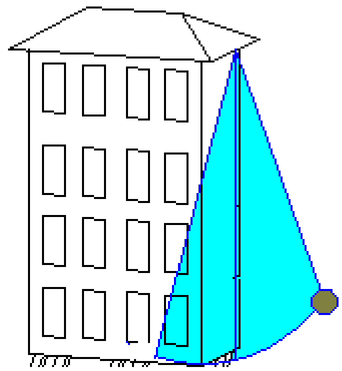
Parallel axes theorem: $I = I_{CM} + m\ell^2$

$$T = 2\pi\sqrt{\frac{M(0.1^2) + M(0.3^2)}{M\times 9.8\times 0.3}} = 2\pi\sqrt{\frac{0.1}{9.8\times 0.3}} = \mathbf{1.16\ s}$$

Problem 4.- If we build a pendulum and we count 20 complete oscillations in 53 seconds, what is the length of the pendulum?

Solution: Since $\mathrm{T} = 2\pi\sqrt{\frac{\mathrm{L}}{\mathrm{g}}}$ then $\mathrm{L} = \frac{gT^2}{4\pi^2} = \frac{9.8(53/20)^2}{4(3.1416)^2} = \mathbf{1.74\ m}$

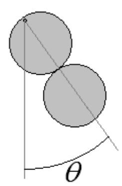
Problem 4a.- A pendulum and a chronometer are used to measure the height of a building as shown in the figure. If the period is 6.95s, how tall is the building?



Solution: Since $\mathrm{T} = 2\pi\sqrt{\frac{\mathrm{L}}{\mathrm{g}}}$ then $\mathrm{L} = \frac{gT^2}{4\pi^2} = \frac{9.8m/s^2(6.95s)^2}{4(3.1416)^2} =$ **12.0 m**

Problem 5.- Consider a pendulum made of two identical disks of radius R=6m as shown in the figure. Calculate the period for small oscillations.

$T = 2\pi\sqrt{\frac{I}{mg\ell}}$ and the moment of inertia of a disk about its center is $I_{CM-disk} = \frac{1}{2}MR^2$

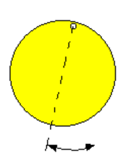


Solution: The total mass is 2M.
The distance from the center of rotation to the center of mass is 2R.

The moment of inertia is $I = \frac{1}{2}MR^2 + MR^2 + \frac{1}{2}MR^2 + 9MR^2 = 11MR^2$

The period is $T = 2\pi\sqrt{\frac{I}{mg\ell}} = 2\pi\sqrt{\frac{11MR^2}{2Mg2R}} = \pi\sqrt{\frac{11R}{g}} =$ **8.2 s**

Problem 6.- A pendulum is made with one solid disk that rotates about a hole located very close to its edge.
Find the period of small oscillations if the radius of the disk is 0.35m
Recall that the moment of inertia of a disk **about its center** is $0.5MR^2$



Solution: The period is $T = 2\pi\sqrt{\frac{I}{mg\ell}} = 2\pi\sqrt{\frac{0.5mR^2 + mR^2}{mgR}} = 2\pi\sqrt{\frac{1.5R}{g}} =$ **1.45 s**

Problem 7.- If you wanted to use a 1m pendulum to measure the gravitational acceleration with 4 significant figures, what kind of precision would you need in the period?

Suggestion: calculate the difference in period for g=9.801m/s^2 and g=9.800m/s^2

Solution: We would need to distinguish between two periods that are very close:

$$T_1 = 2\pi\sqrt{\frac{L}{g}} = 2(3.1415926)\sqrt{\frac{1.0000\text{m}}{9.800\text{m/s}^2}} = 2.00709 \text{ s}$$

$$T_2 = 2\pi\sqrt{\frac{L}{g}} = 2(3.1415926)\sqrt{\frac{1.0000\text{m}}{9.801\text{m/s}^2}} = 2.00699 \text{ s}$$

So T1-T2 = 0.0001s, which means that we would need tenths of millisecond accuracy.

Problem 8.- A pendulum clock was designed to have a period of 1.50 seconds, but due to low quality of its manufacturing its length stretches 0.012 meters beyond the designed value. How long is the actual period?

Solution: The period of a pendulum is: $T = 2\pi\sqrt{\frac{L}{g}}$

The equation with the correct length is: $1.5 = 2\pi\sqrt{\frac{L}{g}}$, so $L = \frac{1.5^2 \times 9.8}{4\pi^2} = 0.5585m$

But the actual length is: $L = 0.558 + 0.012 = 0.5705m$

So, the actual period is: $T = 2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{0.5705}{9.8}} =$ **1.52 s**

Problem 9.- How long must a simple pendulum be if it is to make one complete vibration (one cycle) in 4.0 s? [Acceleration of gravity = 9.8 m/s^2]

Solution: We can use the formula $\mathrm{T} = 2\pi\sqrt{\frac{\mathrm{L}}{\mathrm{g}}}$ to solve for L:

Squaring both sides of the equation: $\mathrm{T}^2 = 4\pi^2\frac{\mathrm{L}}{\mathrm{g}}$

Solving for L: $L = \frac{T^2 g}{4\pi^2} = \frac{(4.0s)^2(9.8m/s^2)}{4(3.1416)^2} =$ **4.0 m**

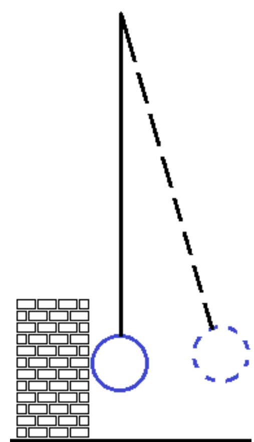
Problem 10.- True (T) or false (F) about the simple pendulum with a small angle of amplitude:

( ) The period depends on the mass of the bob
( ) If you double the length of the pendulum, the period doubles.
( ) If you double the angle of amplitude, the period stays almost the same.

Solution:

(**F**) The period depends on the mass of the bob
(**F**) If you double the length of the pendulum, the period doubles.
(**T**) If you double the angle of amplitude, the period stays almost the same.

Problem 11.- A wrecking ball is hanging from a 16m long steel cable and almost touching a wall to be demolished. It is pulled a short distance from the vertical line and let go from rest. Calculate the time it will take to reach the wall.

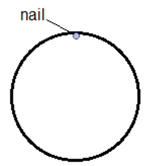


Solution: If we consider the wrecking ball with its cable as a pendulum its period would be:

$$T = 2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{16}{9.8}} = 8.03s$$

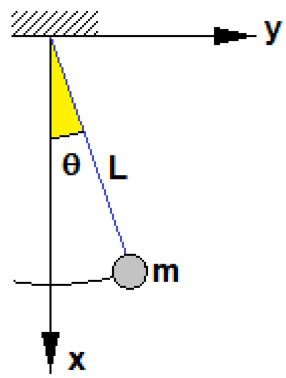
The time to get from one end of the swing to the middle point is only ¼ of that, so **2.0s**

Problem 12.- A hoop of radius R=0.3m is hanging from a nail. You give the hoop a nudge and it starts oscillating like a pendulum. Calculate the period.
Moment of a hoop about its center = MR^2
Parallel Axes theorem: $I = I_{CM} + M\ell^2$



Solution: The period is $T = 2\pi\sqrt{\dfrac{I}{mg\ell}} = 2\pi\sqrt{\dfrac{mR^2 + mR^2}{mgR}} = 2\pi\sqrt{\dfrac{2R}{g}} =$ **1.55 s**

Problem 13.- A pendulum oscillates back and forth around the vertical direction. Its motion starts at the lowest point, and it moves initially to the right. The angle has direction +k when it is to the right of the vertical line and –k when it is on the left.
You are given three graphs that describe the motion of the bob: angle, angular velocity and angular acceleration vs. time.

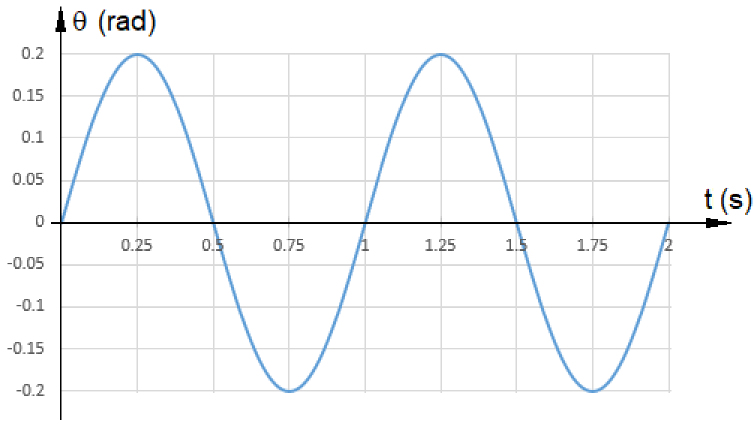


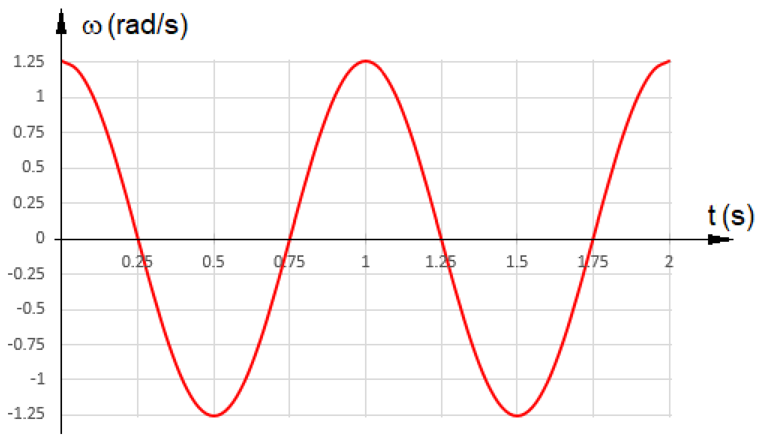
a) Determine the maximum angular displacement with respect to the vertical and the time it takes to get back to this point.

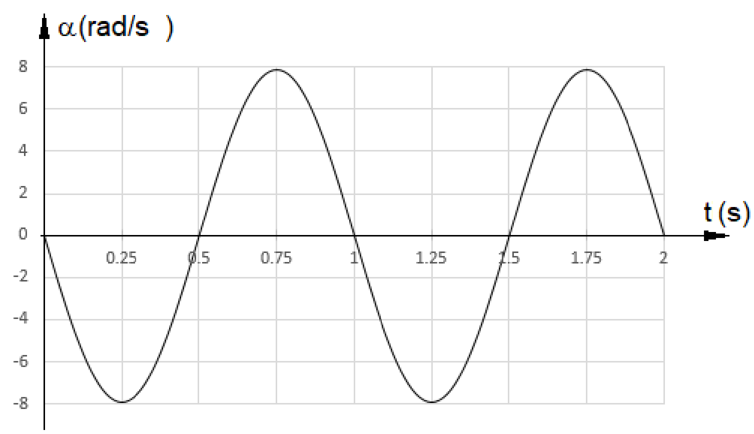
b) What is the average angular velocity and acceleration between 0.25s and 0.5s?

c) Between 0 and 0.5s there are two instants when the angle is 0.1 radians +z, but in one case the angular velocity points to +z and in the other to –z. Why is it that the angular velocity does not have the same direction as the angle?

d) Knowing that the length of the string is L = 0.25m. Find the maximum tangential acceleration and the maximum radial acceleration.







Solution:

a) By observing the graph of angle vs. time we notice that the maximum oscillation (amplitude) is **0.2 radians**. The time needed to return to that point is one period, from the graph we see it is **1 second**.

b) By definition of average angular velocity, we take the final angle minus the initial and divide the difference by the time:

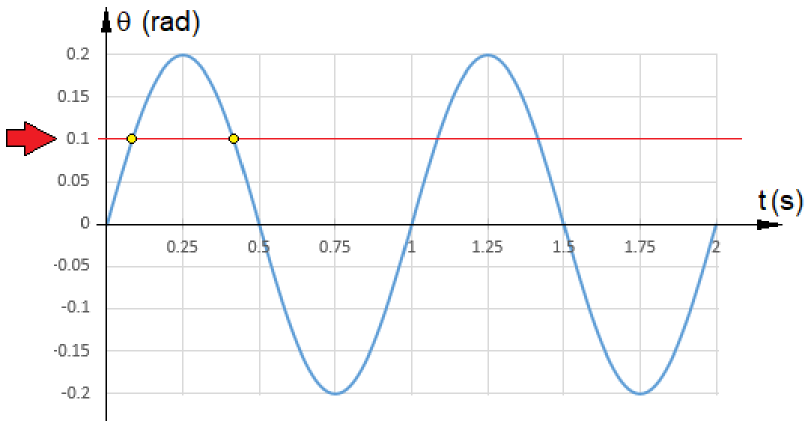
ω = (0-0.2rad)/(0.5s-0.25s) = **-0.8 rad/s (k)**

We notice this is negative (it is in the –k direction).

Similarly, for the acceleration, we use the definition of final minus initial angular velocities, divided by the time:

α = (-1.26rad/s-0)/(05s-0.25s) = **- 5.04 rad/s^2 (k)**

c) The points that the question refers to are shown below.



Indeed, the angle 0.1rad is reached twice in the interval 0 to 0.5s, but in the first case the angle is increasing, indicating a positive angular velocity, and in the second it is decreasing, indicating a negative angular velocity.

d) In cases of circular trajectories, we can calculate the tangential acceleration multiplying the radius by the angular acceleration. To obtain the maximum we take the value from the graph, which is 8.0rad/s^2.

$a_{tangential} = L\times\alpha_{max}$ =0.25m×8.0rad/s^2 = **2.0m/s^2**

It is interesting to note that the first maximum occurs at t=0.75s, which is the extreme left of the oscillation, when the angular velocity is zero.

On the other hand, the radial acceleration can be calculated taking the radius times the angular velocity squared. The maximum is:

$a_{radial} = L\times\omega^2_{max}$ =0.25m×(1.26rad/s)^2 = **0.395m/s^2**

Where we used 1.26 rad/s taken from the graph.