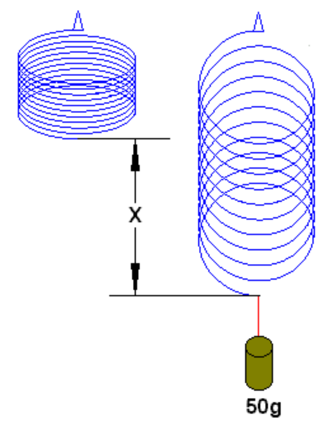
Physics I

Simple Harmonic Oscillator

$F = -kx$ Hooke's law

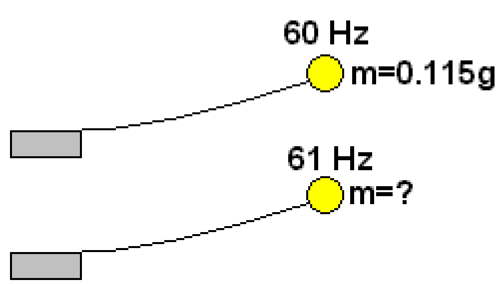
$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$ and $T = 2\pi\sqrt{\frac{m}{k}}$ for a simple harmonic oscillator

Problem 1.- You hang a 50gram mass from a spring and it stretches x = 25cm, then you pull the mass 5 cm from equilibrium and release it. Calculate the period of the oscillation.
Ignore the mass of the spring.



Problem 1a.- A fisherman's scale stretches 2.5cm when a 2.5-kg fish hangs from it.
(a) Calculate the spring constant of the scale in N/m.
(b) What will be the frequency of oscillation if you pull the fish down and release it?

Problem 2.- A frequency indicator has a spring leaf of negligible mass itself with a mass of 0.115 grams attached at its end. Its natural oscillation frequency in this way is 60.0 Hz.
Calculate the mass that you would need for 61.0 Hz.
Hint: Notice that "k" is the same in both cases.



Problem 3.- True (T) or false (F) about the simple harmonic oscillator:

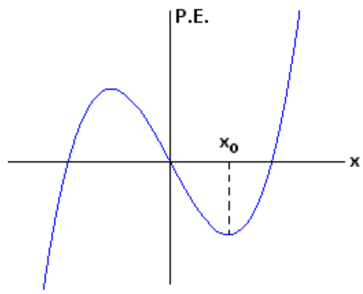
( ) If you double the amplitude, the period doubles.
( ) If you double the mass, the period doubles.
( ) If you quadruple the mass, the period doubles.
( ) If you increase the stiffness of the spring (increase k), the period decreases.
( ) The acceleration in the middle is zero.

Problem 4.- At what displacement x will a simple harmonic oscillator reach 80% of its maximum velocity? Give your answer as a fraction of the amplitude A.

Problem 4a.- What is the velocity of a simple harmonic oscillator when its position is one third the amplitude ($x=A/3$)? Give your answer as a fraction of the maximum velocity.

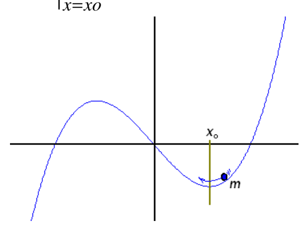
Problem 5.- A particle of mass **m=2kg** is trapped in the potential: $\text{P.E.} = x^3 - 4x$
Find the value of x_o, where the potential has a local minimum and find the period of small oscillations of the particle around that point.

[Remember that $\left.\frac{d^2\text{P.E.}}{dx^2}\right|_{x_o}$ plays the role of "k" in the simple harmonic oscillator]



Problem 5a.- The potential:
$V = x^3 - 3x$
Where V is in joules and x in meters, has a minimum at $x_o = 1$
A particle of mass $m=1kg$ is trapped by the potential. Find the angular frequency of small oscillations around x_o

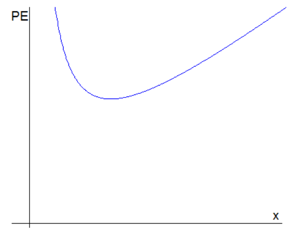
Recall that $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$ *and* $k = \left.\frac{d^2V}{dx^2}\right|_{x=xo}$



Problem 5b.- A particle of mass **m=0.48kg** is trapped in the potential: $P.E. = 4x + \frac{9}{x}$

Find the value of x_o, where the potential has a local minimum and find the period of small oscillations of the particle around that point.

[Remember that $\left.\frac{d^2\text{P.E.}}{dx^2}\right|_{x_o}$ plays the role of "k" in the simple harmonic oscillator]



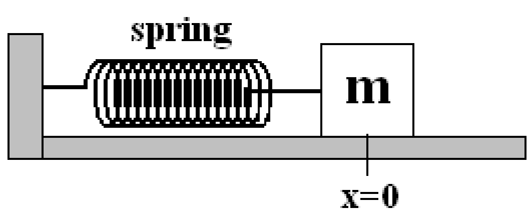
Problem 6.- The vibrational frequency of H_2 is 1.32×10^{14} Hz. So, how much is the vibrational frequency of D_2 (deuterium) that has the same "spring constant" but twice the mass?

Problem 6a.- The vibrational frequency of H_2 is 1.32×10^{14} Hz. So, how much is the vibrational frequency of T_2 (tritium) that has the same "spring constant" but three times the mass?

Problem 7.- A supermarket's scale in the produce section stretches 5.5cm when a 2.5-kg watermelon is added to the plate. If the mass of the plate is 0.5kg:
(a) Calculate the spring constant of the scale in N/m.
(b) What will be the frequency of oscillation if you push the watermelon down and release it?

Problem 7a.- The springs of an 800-kg car compress 5 mm when a 75-kg driver gets into the driver's seat. Calculate the frequency of vibration after hitting a bump (assume there are no shock absorbers).

Problem 8.- The mass shown in the figure, which was in equilibrium is pulled to the right and released. There is negligible friction between the block and the surface, so the mass oscillates back and forth.



a) Where is the acceleration of the block maximum?

b) Where is the velocity of the block maximum?

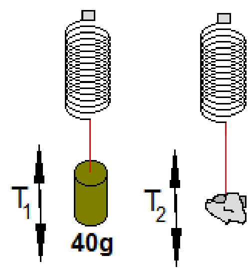
c) Is the acceleration of the block ever zero? if so, where?

d) Is the velocity of the block ever zero? if so, where?

Problem 9.- A bungee jumper of mass 60 kg jumps from a high bridge and oscillates up and down with a frequency of 0.5 Hz. Calculate the frequency of oscillation for another jumper of mass 80 kg that uses the same bungee cord.

Problem 9a.- A fly of mass 0.025g is caught in a spider's web. The web vibrates with a frequency of 4.5Hz. What would be the frequency if the mass of the fly were 0.049g? Assume the web behaves like an ideal spring, making this a case of simple harmonic motion.

Problem 10.- In the lab you hang 40 grams from a spring and it oscillates with a period T_1=1.2 seconds, then you replace the mass with an unknown rock and the period is now T_2=0.6 seconds. What is the mass of the rock?



Problem 11.- An industrial separator shakes coffee beans by vibrating a table vertically following the equation $y = 0.15 \sin \omega t$

Calculate the value of ω, so the beans lose contact with the table.

Problem 12.- The position of a simple harmonic oscillator is given by the equation:

$$x = 5.5\,\mathrm{Sin}(4\pi t)$$

Where x is in meters and t is in seconds.

(a) Calculate the period and frequency

(b) What will be the velocity and acceleration at t = 2.0s?

Problem 13.- The position of a simple harmonic oscillator is given by the equation:

$$x = 8.5 \sin(4\pi t)$$

Where x is in meters and t is in seconds.

Calculate:

(a) Period

(b) Frequency

(c) Maximum speed

(d) Maximum acceleration