Physics I

Gravitation

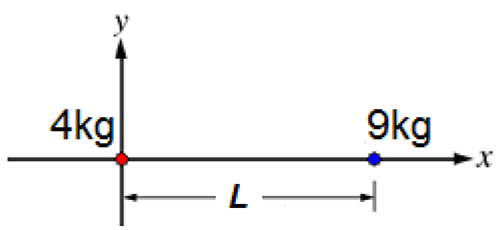
Force between two bodies of mass m_1 and m_2: $F = G\frac{m_1 m_2}{r^2}$, where $G = 6.67\times10^{-11}\frac{Nm^2}{kg^2}$

Acceleration due to gravity on the surface of a planet: $g_{planet} = G\frac{m_{planet}}{r_{planet}^2}$

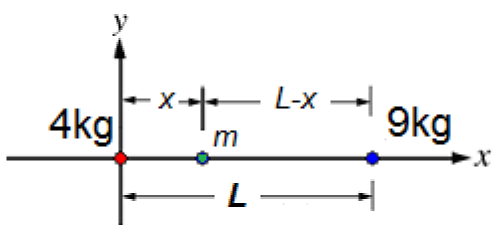
Gravitational potential energy $P.E. = mgh$ … close to the surface of the Earth

But $P.E. = -G\frac{Mm}{r}$ is more general, with the reference is at infinity.

Problem 1.- A mass of 4kg is placed at the origin and a mass of 9kg is placed at ***L***=+15 meters as shown in the figure. At what point on the x-axis will a test mass of ***m*** kilograms experience zero net force?



Solution: Suppose the third mass ***m*** is at a position *x* from the 4kg mass, then the distance to the 9kg mass will be 15-*x* and the two attractive forces have to be equal to be in equilibrium, so:



$$G\frac{4m}{x^2} = G\frac{9m}{(15-x)^2} \rightarrow \frac{4}{x^2} = \frac{9}{(15-x)^2} \rightarrow \frac{2}{x} = \frac{3}{15-x} \rightarrow 30-2x = 3x \rightarrow x = \mathbf{6m}$$

Problem 2.- Calculate the acceleration due to gravity on the surface of Ganymede (a satellite of Jupiter visible in binoculars), which has 0.0248 times the mass of the Earth and whose radius is 0.413 times the radius of the Earth.

Solution: For Ganymede:

$$g_{Ganymede} = G\frac{m_{Ganymede}}{r_{Ganymede}^2} = G\frac{(0.0248m_{Earth})}{(0.413r_{Earth})^2} = \frac{(0.0248)}{(0.413)^2}\left(G\frac{m_{Earth}}{r_{Earth}^2}\right)$$

But the quantity in parenthesis is our beloved g=9.8m/s^2, so:

$$g_{Ganymede} = \frac{(0.0248)}{(0.413)^2}\left(9.8m/s^2\right) = \mathbf{1.43\ m/s^2}$$

Problem 2a.- Calculate the acceleration due to gravity on the surface of Ceres (g_{ceres}). Ceres is one of the largest known asteroids in the solar system. For your calculation assume the asteroid has the shape of a sphere with radius 460 km and mass 7.4×10^{21} kg.

Solution: Let us use the equation: $\mathrm{g}_{\mathrm{planet}} = \mathrm{G}\dfrac{\mathrm{m}_{\mathrm{planet}}}{\mathrm{r}^2_{\mathrm{planet}}}$

With the values given, we get:

$$g_{Ceres} = 6.67\times10^{-11}\frac{Nm^2}{kg^2}\frac{7.4\times10^{21}kg}{(460{,}000m)^2} = \mathbf{2.33\ m/s^2}$$

Problem 2c.- Calculate the acceleration due to gravity on the surface of Mars, which has 0.11 times the mass of the Earth and whose radius is 0.53 times the radius of the Earth.

Solution:

For Mars: $$g_{Mars} = G\frac{m_{Mars}}{r^2_{Mars}} = G\frac{(0.11m_{Earth})}{(0.53r_{Earth})^2} = \frac{(0.11)}{(0.53)^2}\left(G\frac{m_{Earth}}{r^2_{Earth}}\right)$$

But the quantity in parenthesis is g=9.8m/s^2,

$$g_{Mars} = \frac{(0.11)}{(0.53)^2}\left(9.8\frac{m}{s^2}\right) = \mathbf{3.84\ m/s^2}$$

For Jupiter: $$g_{Jupiter} = G\frac{m_{Jupiter}}{r^2_{Jupiter}} = G\frac{(318m_{Earth})}{(10.97r_{Earth})^2} = \frac{(318)}{(10.97)^2}\left(G\frac{m_{Earth}}{r^2_{Earth}}\right)$$

$$g_{Jupiter} = \frac{(318)}{(10.97)^2}\left(9.8\frac{m}{s^2}\right) = \mathbf{25.9\ m/s^2}$$

For Venus: $$g_{Venus} = G\frac{m_{Venus}}{r^2_{Venus}} = G\frac{(0.88m_{Earth})}{(0.95r_{Earth})^2} = \frac{(0.88)}{(0.95)^2}\left(G\frac{m_{Earth}}{r^2_{Earth}}\right)$$

$$g_{Venus} = \frac{(0.88)}{(0.95)^2}\left(9.8\frac{m}{s^2}\right) = \mathbf{9.56\ m/s^2}$$

For Mercury: $g_{Mercury} = G\frac{m_{Mercury}}{r_{Mercury}^2} = G\frac{(0.0553m_{Earth})}{(0.383r_{Earth})^2} = \frac{(0.0553)}{(0.383)^2}\left(G\frac{m_{Earth}}{r_{Earth}^2}\right)$

$g_{Mercury} = \frac{(0.0553)}{(0.383)^2}\left(9.8\frac{m}{s^2}\right) =$ **3.7 m/s²**

Problem 2d.- What is the acceleration due to gravity on the surface of Pluto given that its radius is 0.18 the radius of the Earth and its mass is 0.0021 the mass of the Earth.

Solution: $g_{pluto} = G\frac{0.0021M_E}{(0.18R_E)^2} = \left(G\frac{M_E}{R_E{}^2}\right)\frac{0.0021}{0.18^2} = (9.8)\frac{0.0021}{0.18^2} =$ **0.635 m/s²**

Problem 3.- What would be the speed of an object that falls straight towards the Earth from a height of $h=4\times10^6$ m when it reaches the surface of our planet?
[Ignore air resistance, assume initial velocity zero]
[Radius of the Earth $R = 6.38\times10^6$ m]

Solution: $PE = KE \rightarrow mgh\left(\frac{R_E{}^2}{R_1R_2}\right) = \frac{1}{2}mv^2 \rightarrow v = \sqrt{2gh\left(\frac{R_E{}^2}{R_1R_2}\right)}$

$\rightarrow v = \sqrt{2\times9.8\times4\times10^6\left(\frac{6.38\times10^6}{6.38\times10^6+4\times10^6}\right)}$ =**6,900 m/s**

Problem 4.- Find the escape velocity from the surface of Mars whose mass is 0.64×10^{24} kg and a radius of 3.39×10^6 m.

Solution Since the total energy is zero to escape:

$\frac{1}{2}mv^2 - G\frac{Mm}{R} = 0 \rightarrow v = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2(6.67\times10^{-11})(0.64\times10^{24})}{3.39\times10^6}}$ = **5,018 m/s**

Problem 4a.- Calculate the escape velocity from the surface of Pluto. For your calculation assume it has the shape of a sphere with radius 1,153 km and mass 1.305×10^{22} kg.

Solution: We use the equation for total energy:

$0 = -G\frac{Mm}{R} + \frac{1}{2}mv^2 \rightarrow v = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2\times6.67\times10^{-11}\times1.305\times10^{22}}{1{,}153{,}000}} =$ **1,228 m/s**

Problem 4b.- Calculate the escape velocity from the surface of Ceres. Ceres is one of the largest known asteroids in the solar system. For your calculation assume the asteroid has the shape of a sphere with radius 460 km and mass 7.4×10^{21} kg.

Solution: If an object is going to leave the asteroid, it must have zero total energy:

$$-G\frac{Mm}{R}+\frac{1}{2}mv^2=0\rightarrow v=\sqrt{\frac{2GM}{R}}=\sqrt{2\times6.67\times10^{-11}\frac{Nm^2}{kg^2}\frac{7.4\times10^{21}kg}{(460000m)}}=\mathbf{1{,}470\ m/s}$$

Problem 5.- Calculate the acceleration due to gravity on the surface of a small planet that has 0.125 times the mass and 0.5 times the radius of our planet.

Solution: $a_R=G\frac{M}{R^2}=G\frac{0.125M_E}{(0.5R_E)^2}=0.5G\frac{M_E}{R_E^2}0.5\times9.8=\mathbf{4.9\ m/s^2}$

Problem 5a.- Calculate the acceleration due to gravity on the surface of a giant planet that has 195 times the mass and 8 times the radius of our planet.

Solution: We want $g_{planet}=G\frac{m_{planet}}{r_{planet}^2}$, given that the mass of the planet is 195 times the mass of Earth and radius 8 the radius of Earth:

$$g_{planet}=G\frac{195m_{Earth}}{(8r_{Earth})^2}=\frac{195}{64}G\frac{m_{Earth}}{(r_{Earth})^2}=\frac{195}{64}g_{Eartth}=\frac{195}{64}(9.8m/s^2)=\mathbf{29.9\ m/s^2}$$

Problem 5b.- Find the escape velocity from the surface of the planet mentioned in the previous problem.

Solution: If the object is going to escape, it should have at least zero total energy:

$$\frac{1}{2}mv^2-G\frac{mM}{R}=0\rightarrow v=\sqrt{\frac{2GM}{R}}=\sqrt{\frac{2(6.67\times10^{-11})(195\times5.98\times10^{24})}{8\times6.38\times10^6}}=\mathbf{55{,}200\ m/s}$$

Problem 6.- Gravity is usually neglected in condensed matter and atomic physics because it is extremely weak compared to electromagnetic forces. To convince yourself that this is true: Calculate the gravitational attraction between an electron and an alpha particle (a helium nucleus) separated by 1 nm. (1 nm is 10^{-9} m)
Mass of the electron = 9.1×10^{-31} kg
Mass of the alpha particle = 6.7×10^{-27} kg

Solution: Gravitational force between an alpha particle and an electron separated 1nm:

$$F = 6.67\times10^{-11}\frac{(9.1\times10^{-31}kg)(6.7\times10^{-27}kg)}{(1\times10^{-9}m)^2} = \mathbf{4.07\times10^{-49}\ N}$$

Problem 7.- Planet Y has twice the value of g as our planet (19.6 m/s^2) and it has 8 times the mass of Earth. What is the radius of planet Y?

Answer in terms of our planet's radius (R_E).

(A) $0.5R_E$ (B) R_E (C) $2R_E$ (D) $4R_E$ (E) $16R_E$

Solution: $2g = G\frac{M_P}{R_P^2}$ and $g = G\frac{M_E}{R_E^2}$,

Dividing one equation by the other:

$$\frac{2g}{g} = \frac{G\frac{M_P}{R_P^2}}{G\frac{M_E}{R_E^2}} \to 2 = \frac{8R_E^2}{R_P^2} \to R_P = 2R_E$$

Problem 8.- Calculate the force due to gravity on a satellite of mass 84kg located 3 Earth radii away from the *surface* of the Earth.

Solution: The force is $F = G\frac{m_1m_2}{d^2}$, but $d = 4R_E$, so $F = G\frac{M_{Earth}84}{16R_E^2} = \frac{84}{16}\times9.8 =$ **51.5 N**

Problem 9.- How far from the surface of the Earth would you have to go to have acceleration due to gravity of only 1/1000 of the value at the Earth surface?

Solution: The force due to gravity is inversely proportional to the distance to the center of the planet, so: $\frac{r_{planet}^2}{x^2} = \frac{1}{1000} \to x = 31.6r_{planet}$

This is the distance between the planet center and the object, so **30.6** radii from the surface.

Problem 10.- It is common to hear that people inside a space station, in orbit around the Earth, experiment "zero gravity". Is the gravity really zero inside the station?

Solution: Not really. People inside the space station are accelerated towards the center of the Earth. In a sense they are falling. For Low Earth Orbits (LEOs) the acceleration due to gravity (g) is ~90% of the value on the surface.