Physics I

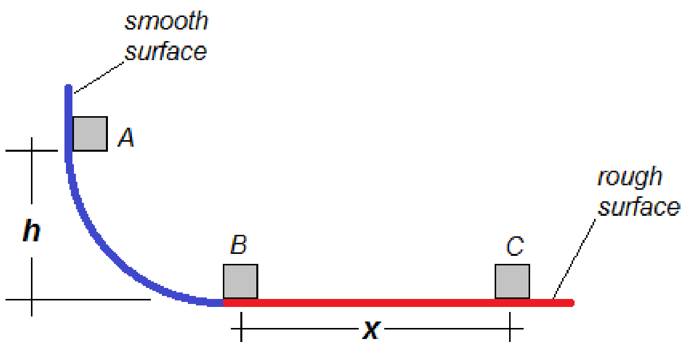
Energy Conservation

Work $W = Fd\cos\angle_F^d$

Kinetic energy linear motion $K.E. = \frac{1}{2}mv^2$

Gravitational potential energy $P.E. = mgh$

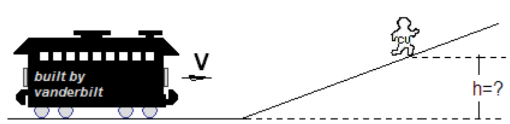
Problem 1.- A block is released from position *A* with zero initial velocity at a height *h* = 4m. It accelerates towards position *B* without friction. At point *B*, it starts to slow down due to friction and stops at *C*.
Calculate the distance *x* if $\mu = 0.4$
You can use the equations $F_{friction} = \mu F_{Normal}$, $F = ma$ and $v_2^{\,2} = v_1^{\,2} + 2ax$



Solution: The potential energy at point A is completely lost by doing work against friction between B and C, so:

$$mgh = F_{friction}x \to mgh = \mu mgx \to x = \frac{h}{\mu} = \frac{4}{0.4} = \textbf{10 m}$$

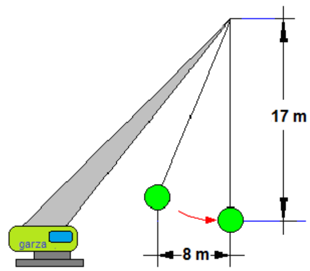
Problem 2.- A loose train car comes toward you at a speed v = 12m/s on a level surface. In order to protect your life, you run up an incline. How far up do you need to go to be safe?
Assume you cannot jump to the side. Ignore friction.



Solution: The train will go up until all its kinetic energy is traded for potential energy, so:
$\frac{1}{2}mv^2 = mgh \to h = \frac{v^2}{2g}$,
Anything higher than that and the train won't reach you.

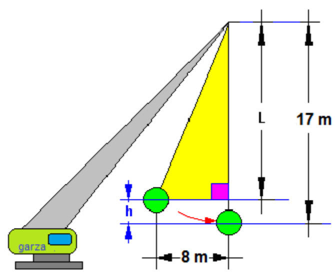
Given the values of the problem: $h = \frac{v^2}{2g} = \frac{(12m/s)^2}{2(9.8m/s^2)} = \textbf{7.3 m}$

Problem 3.- A wrecking ball has a mass of 850 kg and is suspended from a crane by a 17m long cable. The ball is pulled 8 m away from the vertical and released. Calculate:
i) The velocity at the bottom of the trajectory
ii) The kinetic energy at that point.



Solution: Notice that the difference in height can be calculated using Pythagoras theorem. The right triangle highlighted in yellow (see below) has a long side of 17 m and one of the short sides is 8m, so the other side is:

$L = \sqrt{17^2 - 8^2} = 15$ so, the height is h=2m

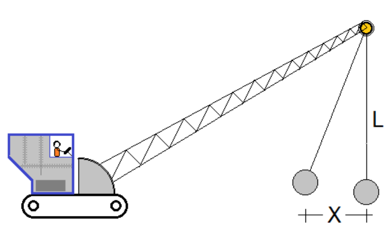


Potential energy is converted into kinetic energy, so:

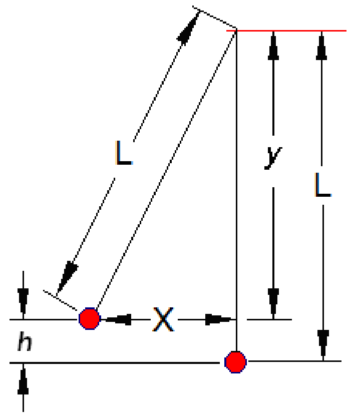
$P.E. = mgh = 850kg \times 9.8\frac{m}{s^2} \times 2m =$ **16,660 J**

To calculate the speed, $mgh = \frac{1}{2}mv^2 \rightarrow v = \sqrt{2gh} = \sqrt{2 \times 9.8 \times 2} =$ **6.3 m/s**

Problem 3a.- A wrecking ball of mass 250 kg is suspended from a crane with a cable of length L= 13m. The ball is pulled a distance X= 5m away from the vertical and released. Calculate the kinetic energy of the ball when it reaches the bottom of the trajectory.



Solution We can calculate the difference in height between the initial and final points. The figure shows the geometry:



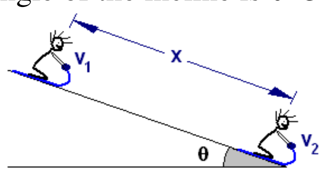
y can be calculated using Pythagoras' theorem:

$y = \sqrt{13^2 - 5^2} = 12$, so, the height is $h = 13 - 12 = 1m$

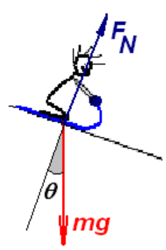
The kinetic energy at the bottom of the trajectory is equal to the change in potential energy:

$mgh = 250 \times 9.8 \times 1 =$ **2,450 J**

Problem 4.- A sled starts down a slope with an initial velocity **v_1=4m/s**. Calculate its final velocity after sliding **x=10m** if the angle of the incline is **θ=30º** and we neglect friction.



Solution: There are only two forces on the sled, namely its weight and the normal force. The vector diagram looks as follows:



Since the normal force does not do any work in this case, we know that the total mechanical energy is the same:

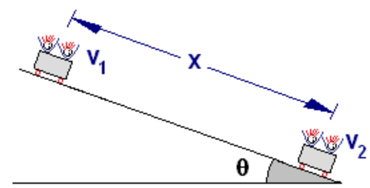
$$K.E._1 + P.E._1 = K.E._2 + P.E._2$$

If we set the reference at the level of point 2, then $P.E._2 = 0$
We also notice that the height of point 1 is: $h = x\sin\theta$, so the equation is:

$$\frac{1}{2}mv_1^{\ 2} + mgx\sin\theta = \frac{1}{2}mv_2^{\ 2}$$

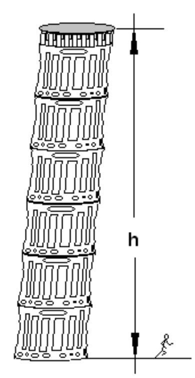
Solving for v_2 we get: $v_2 = \sqrt{v_1^{\,2} + 2gx\sin\theta} = \sqrt{4^2 + 2(9.8)(10)\sin 30^\circ}$ $=$ **11 m/s**

Problem 4a.- A rollercoaster car starts down a slope with an initial velocity ***v*₁=4m/s**. Calculate its final velocity after a distance ***x*=5m** along the slope if the angle of the incline is **θ=37º** and we neglect friction.



Solution: $\frac{1}{2}v_2^{\,2} = \frac{1}{2}4^2 + 9.8\times 5\sin 37^\circ \rightarrow v_2 =$ **8.7 m/s**

Problem 5.- A leaning tower of height h=34 m finally falls. Calculate the speed of the tip of the tower when it hits the ground.
To solve the problem, ignore the initial leaning angle, assume that the mass of the tower is uniformly distributed, ignore any external torque and approximate the moment of inertia to the one of a rod rotated about one end $\mathrm{I} = \frac{1}{3}\mathrm{ML}^2$



Solution: We can solve this problem by considering all the potential energy being converted into rotational kinetic energy:

$$\mathrm{mgh} = \frac{1}{2}\mathrm{I}\omega^2$$

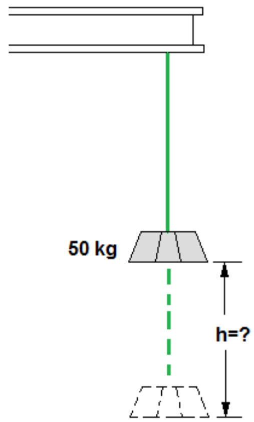
There is an important detail here, the center of mass of the tower is located at only half the height (at L/2), then:

$$\mathrm{mg}\frac{\mathrm{L}}{2} = \frac{1}{2}\left(\frac{1}{3}\mathrm{m}\times\mathrm{L}^2\right)\omega^2 \rightarrow \omega = \sqrt{\frac{3\mathrm{g}}{\mathrm{L}}}$$

And to find the speed:

$$v = \omega r = \sqrt{\frac{3g}{L}}L = \sqrt{3gL} = \sqrt{3\times 9.8\times 34} = \mathbf{32\ m/s}$$

Problem 6.- Certain bungee cord behaves like a spring with constant k=800 N/m.
Suppose you attach a 50-kg mass to the end of the un-stretched cord that is hanging from a beam and release it (with initial v=0). How far down will the mass go?



Solution: Notice that the gravitational potential energy lost will be converted into potential energy stored in the spring stretching:

$$mgh = \frac{1}{2}kh^2 \rightarrow h = \frac{2mg}{k} = \frac{2\times 50\times 9.8}{800} = \mathbf{1.2\ m}$$

Problem 7.- A 4g projectile traveling at 450 m/s hits a thick window and its stops after traveling 0.076m in the material. Find the average force on the projectile and how much kinetic energy was lost.

Solution: All the kinetic energy is lost because the projectile stops, so:

$$K.E. = \frac{1}{2}mv^2 = \frac{1}{2}(0.004kg)(450m/s)^2 = \mathbf{405\ J}$$

This kinetic energy is equal to the work done by the force:

$$K.E. = Fd \rightarrow F = KE/d = 405J/0.076m = \mathbf{5{,}300\ N}$$

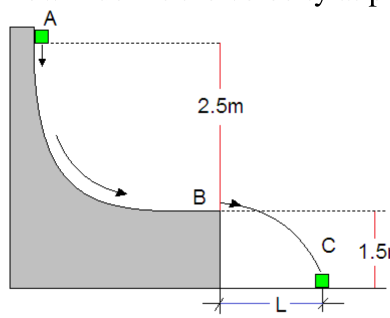
Problem 8.- A horizontal force of 230 N is applied to move a 68-kg cart (initially at rest) across a 13m-long level surface. What is the final speed of the cart?
Ignore friction in this problem.

Solution: The acceleration will be $a = \frac{F}{m} = \frac{230}{68} = 3.38m/s^2$

We know the initial velocity (zero), so the final velocity will be:

$$v_2^{\ 2} = v_1^{\ 2} + 2ax = 0^2 + 2\times 3.38\times 13 \rightarrow v_2 = \sqrt{2\times 3.38\times 13} = \mathbf{9.4\ m/s}$$

Problem 9.- A solid block is released from point A. It slides without friction towards point B and finally lands at C. Assume the block is released from A with zero initial velocity, and its velocity at point B is horizontal. How much is the velocity at point B?



Solution: To get the velocity at point B we use the fact that the potential energy is converted into kinetic energy:

$$mgh = \frac{1}{2}mv^2$$

$$v = \sqrt{2gh} = \sqrt{2\left(9.8\frac{m}{s^2}\right)(4.5m)} = \mathbf{9.4\ m/s}$$

Problem 10.- Calculate the distance L in the problem above.

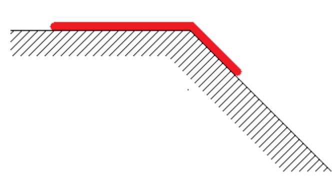
Solution: Notice that the initial velocity is in the horizontal direction, so we can calculate the time it takes to fall using the equation:

$$y = \frac{1}{2}gt^2 \quad \text{(because } v_{1y} = 0\text{), so } t = \sqrt{\frac{2y}{g}} = \sqrt{\frac{2(2.5m)}{9.8m/s^2}} = \mathbf{0.71\ s}$$

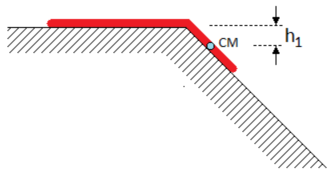
This allows us to find L:

$$L = v_{1x}t = 9.4m/s(0.71s) = \mathbf{6.7\ m}$$

Problem 11.- Consider a uniform chain in the position shown in the figure. The length on the 45º inclined plane is 10 m and 20 m on the leveled surface. Its initial velocity is zero.
Calculate the velocity of the chain when the last link is on the inclined plane. Ignore friction and assume that all the links always stay in contact with the surface (this could be assured by using an elbow at the corner or some other way).



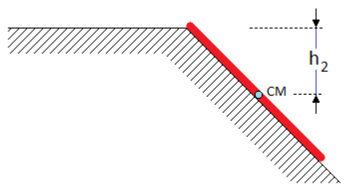
Solution: The potential energy lost by the chain will be converted to kinetic energy. To calculate the change in potential energy we can use the level surface as reference. Initially 2/3 of the mass of the chain is at that reference level and 1/3 of its mass is on the incline surface. The center of mass of that part of the chain in the middle:



The potential energy at that point is:

$$PE_1 = -\frac{m}{3} g h_1 = -\frac{m}{3} g(5\sin 45^\circ)$$

Once the whole chain is in the inclined surface, the center of mass will be in the middle of the whole length:



The potential energy then will be:

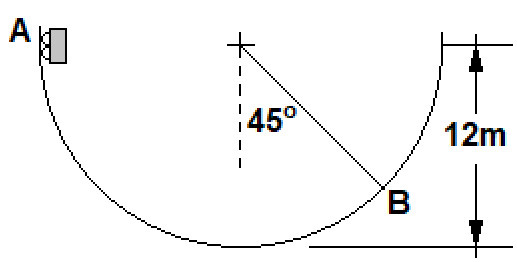
$$PE_2 = -mgh_2 = -mg(15\sin 45^\circ)$$

We can now calculate the final speed of the chain:

$$\frac{1}{2}mv^2 = PE_1 - PE_2 = -\frac{m}{3} g(5\sin 45^\circ) + mg(15\sin 45^\circ) = \frac{mg}{3} 40\sin 45^\circ$$

$$v = \sqrt{\frac{80\sin 45^\circ}{3} g} = \mathbf{13.6m/s}$$

Problem 12.- A small object slides on a semicircular surface without friction. If it starts at point A with an initial velocity of 7m/s, what is its speed at point B?



Solution: The potential energy lost by the small object will be converted to kinetic energy. To calculate the change in potential energy we can use point A as reference. The potential energy at B will be:

$$PE_B = -mg12\sin 45^\circ$$

Accordingly:

$$\frac{1}{2}m{v_B}^2 - \frac{1}{2}m{v_A}^2 = PE_A - PE_B$$

$$\frac{1}{2}m{v_B}^2 - \frac{1}{2}m(7)^2 = 0 - (-mg12\sin 45^\circ)$$

$${v_B}^2 = \sqrt{24\sin 45^\circ g + 7^2} = \textbf{14.7m/s}$$